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A contact dynamics approach to the Granular Element Method

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ABSTRACT

We present a contact dynamics (CD) approach to the Granular Element Method (GEM) Andrade et al. (2012) [1], abbreviated here as CD–GEM. By combining particle shape flexibility through Non-Uniform Rational Basis Splines, properties of implicit time-integration discretization (e.g., larger time steps) and non-penetrating constraints, as well as a reduction to a static formulation in the limit of an infinite time step, CD–GEM targets system properties and deformation regimes in which the classical discrete element method either performs poorly or simply fails; namely, in granular systems comprising of rigid or highly stiff angular particles and subjected to quasi-static or intense dynamic flow conditions. The integration of CD and GEM is made possible while significantly simplifying implementation and maintaining comparable performance with existing CD approaches.

1. Introduction

The objective of this paper is to develop a contact dynamics (CD) approach to the Granular Element Method (GEM) [1], abbreviated here as CD–GEM. CD–GEM targets system properties and deformation regimes in which the classical discrete element method (DEM) either performs poorly or simply fails; namely, in granular systems comprising of rigid or highly stiff angular particles and subjected to quasi-static or intense dynamic flow conditions. Within the context of such applications, we describe how CD–GEM offers a better solution in terms of particle morphology or shape representation and ease of implementation, while maintaining comparable performance with existing CD approaches. To motivate the development of our approach, we first refer the reader to Table 1 for a brief summary of the key features of and differences between CD and the classical DEM by Cundall and Strack [2]. In the following, we highlight the difficulties associated with CD and DEM followed by a description on how we eliminate them through CD–GEM.

The so-called Non-Smooth CD, originally developed by Moreau [6–9], is an alternative discrete approach to the DEM. The most prominent feature of CD, in contrast to that of classical DEM, is that the particles are considered perfectly rigid and the contact forces are determined as those that prevent interparticle penetration and at the same time satisfy the frictional stick–slip constraints. In their simplest forms, these contact laws are embodied in the so-called Signorini unilateral contact condition and classical Coulomb law, as shown in Fig. 1(a) and (b), respectively. Commensurate with these physical enhancements, however, is the need for both contact and constraint forces to be solved simultaneously or implicitly since the problem is nonlinear. The need for an implicit solution procedure till today remains the primary reason why CD is deemed much more complicated to implement than DEM. This has thwarted the wide adoption of CD despite the favorable performance that has been shown through a number of studies [10–20].

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Table 1
Comparison of non-smooth contact dynamics and classical DEM.

Feature	N-S contact dynamics	Classic DEM
Normal contact	Rigid; unilateral contact[1],[2] or non-penetration constraint directly included	Modeled using normal spring; particles overlap
Friction contact	Stick–slip frictional constraint[1,2] directly included	Imposes shear force incrementally using relative velocity from previous step
Time integration	Implicit, usually stable and with larger time step[3]	Explicit, with stability criterion; critical time step scales with inverse of spring frequency. Inefficient for highly stiff particles and cannot be applied to rigid particles
Collision response	Considers collisions and stick–slip frictional transitions simultaneously; velocities may be non-smooth	No real collisions and velocity jumps cannot occur due to continuous nature of contact spring
Damping	Numerical damping[4]	Through global and/or local damping devices, i.e., dashpots
Quasistatic limit	Can be directly included in formulation	Dynamic in nature; oscillations in solutions are typical; quasistatic limit is approached using global and/or local damping
Particle morphology representation[5,6]	Disc- or sphere-clustering and polyhedra	
Implementation difficulty	Intermediate to difficult[7]	Easy
Computational efficiency	Contact and constraint forces solved implicitly. Geometrical information (e.g., gap values and contact orientations) are stored in matrices as part of the solution procedure; higher memory requirement[8]	Contact forces are solved explicitly using particle overlap and previous velocities; time integration easily parallelized. Minimal storage of geometrical information; lower memory requirement

[1] Regularization to account for particle elasticity possible (see for e.g., [3]).
[2] See Fig. 1.
[3] Although the time step can be larger, it has to be reasonable so that collisions are properly resolved.
[4] Does not apply in the quasistatic limit.
[5] We list only those approaches, beyond ellipses/ellipsoids, that appear to be currently most widely applied.
[6] Improved using NURBS in CD–GEM.
[7] Made easier in CD–GEM.
[8] Managed in CD–GEM using efficient large-scale mathematical programming solvers (for e.g., [4,5]).

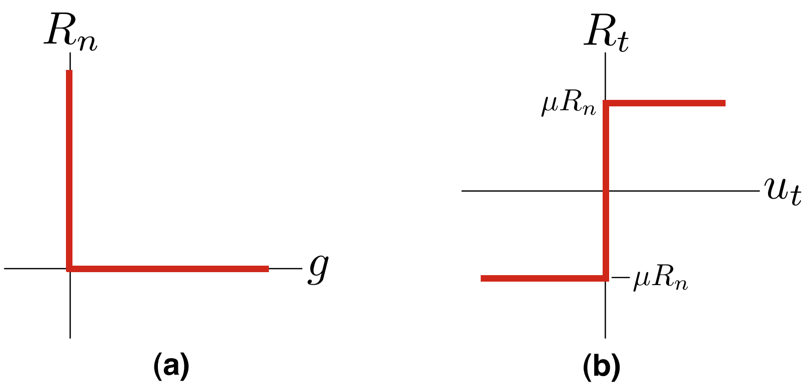

Fig. 1. Graph of non-smooth contact laws: (a) normal reaction force R_n against separation or gap g and (b) friction force R_t against slip displacement u_t; μ is the friction coefficient.

While there is wide applicability of DEM, its application has gone beyond its restriction as a tool that is strictly applicable only to materials with finite elasticity. For example, DEM is widely used as a tool to study real granular materials that are almost rigid or highly stiff in nature. Here, finite elasticity means that the contact interaction is essentially modeled using springs. Under explicit time integration algorithms that are typically used in DEM, the stable time step is restricted by the critical time step, which scales with the inverse of the contact spring-particle mass frequency. This results in infinitesimally small time steps if material parameters corresponding to highly stiff particles (e.g., rocks, sand, steel) are used. Although explicit integration algorithms can be easily parallelized, the runtime for stiff systems remains computationally prohibitive. One modeling technique commonly employed in practice to overcome this restriction is to simply reduce the contact stiffness, usually by two to four orders of magnitude, to the extent that particle kinematics obtained from simulations are still somewhat representative of the overall response of the actual system of interest. If quasi-static behavior is assumed to hold, usually