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Efficient solver implementation for reynolds equation with mass-conserving cavitation

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ABSTRACT

A global grid refinement solver implementation for the Iso-Viscous-Rigid Reynolds equation with cavitation (mass-conservation) using the Fischer-Burmeister equation for complementarity is presented and shows a quasi linear time complexity. The global grid refinement strategy allows a fast and stable convergence. It is applied to several examples of dimple textured flat/parallel surfaces in order to, first illustrate the algorithm performance, and second, to point out discretisation error issues which may occur for textured surfaces and justify the need of an efficient numerical method to solve such cases.

1. Introduction

Power loss and lubricant consumption are the first targets of engine optimisation. A typical example is the cylinder liner - piston ring contact. In addition to ‘classical’ design parameters such as the ring tension, ring width, lubricant viscosity, etc. surface texturing is used [1–3]. Multigrid calculation techniques have been adapted to the textured Iso-Viscous-Rigid (IVR) problem [4]. When a micro-geometry (such as the texture) is considered, cavitation [5–8] may occur inside the contact zone contrary to the classical case where a single cavitation boundary occurs at the contact outlet. In this case, a mass conserving algorithm has to be used in order to avoid lubricant ‘generation’ [9,10]. The purpose of this paper is to present for the simplest conditions (steady-state Iso-ViscousRigid), the implementation of a simple and efficient mass conserving algorithm for the Reynolds equation with cavitation. Several applications to dimple textured flat/parallel surfaces are presented similarly to [11]. Some qualitative trends are extrapolated from these examples. These examples have two objectives: first, they are used to test the algorithm performance in terms of calculation time/complexity, and second, they justify the need of an efficient numerical method such as the one presented in order to solve fine mesh problems which are required to avoid discretisation error issues.

The current paper originates from the work of Woloszynski et al. [12] who present the joint resolution of Reynolds and Fischer-Burmeister (complementarity) equations [13] and benchmark its efficiency against other methods. Here, the focus is on a very simple implementation (around 40 Matlab lines) using the P minus θ combined unknown. This combined unknown has two main advantages: first, the unknowns are (almost) continuous, second, the data management is trivial and no switches between unknowns in P or θ are required. A grid refinement strategy is used to converge the cavitation boundary using an almost constant (and limited) number of iterations. This grid refinement (from coarse to fine) is not a Multigrid strategy where fine levels communicate with coarse ones. It only looks like the Full-Multi-Grid strategy where the problem is solved on finer and finer meshes. The advantage of the grid refinement strategy is that the pressurized zone frontier has only very limited displacements on every level which significantly stabilizes and accelerates the convergence. Alternative mass conserving algorithms for the Reynolds equation with cavitation may use finite elements [14] and [15], or finite volume techniques [16].

2. Method

Two equations are solved simultaneously: the Reynolds equation and the Fischer-Burmeister equation (complementarity). H is the film thickness, P is the pressure, X is the sliding direction, Y is the direction perpendicular to the sliding direction, θ is the void fraction.

$$\frac{\partial}{\partial X}\left(H^3\frac{\partial P}{\partial X}\right)+\frac{\partial}{\partial Y}\left(H^3\frac{\partial P}{\partial Y}\right)-\frac{\partial((1-\theta)H)}{\partial X}=0 \tag{1}$$

$$P+\theta-\sqrt{P^2+\theta^2}=0 \tag{2}$$

Using finite differences, these two equations are discretised

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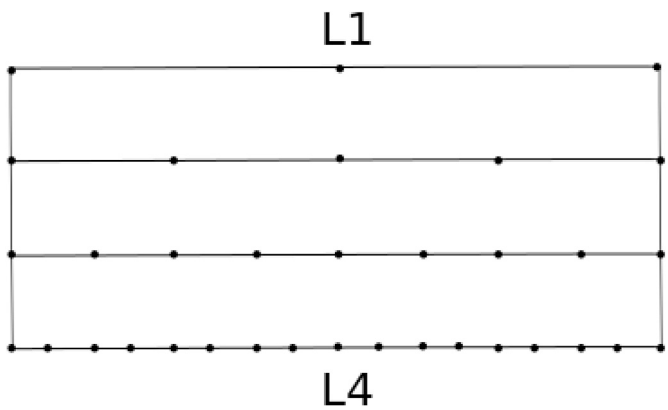

Fig. 1. 1D global grid refinement example.

(assuming a constant mesh size Δ, N points in each direction if the domain is square) and a local linearisation are used (jacobian coefficients are calculated from the partial derivatives of equations (1) and (2) with respect to P_{ij} and θ_{ij}). r are the residuals of equations (1) and (2) and δ are the corrections to be applied to the unknowns P and θ. For IVR contacts, A_P and A_θ are calculated only once because they only depend on the geometry. k designates an index in the jacobian matrices (A and C) whereas i and j designate respectively the point indexes in the X and Y directions.

The equations (3)–(11) describe how the system is solved. The final system which is used is written in equation (11); this is a square N^2xN^2 system. On the left hand side there is the system matrix times a correction vector δ; on the right hand side, there is a residual vector r. To define the system matrix, four intermediate matrices are used: A_P, A_θ (corresponding to an upwind discretisation for a positive speed), C_P, C_θ. These four matrices are defined in equations (3)–(8). However, each matrix has

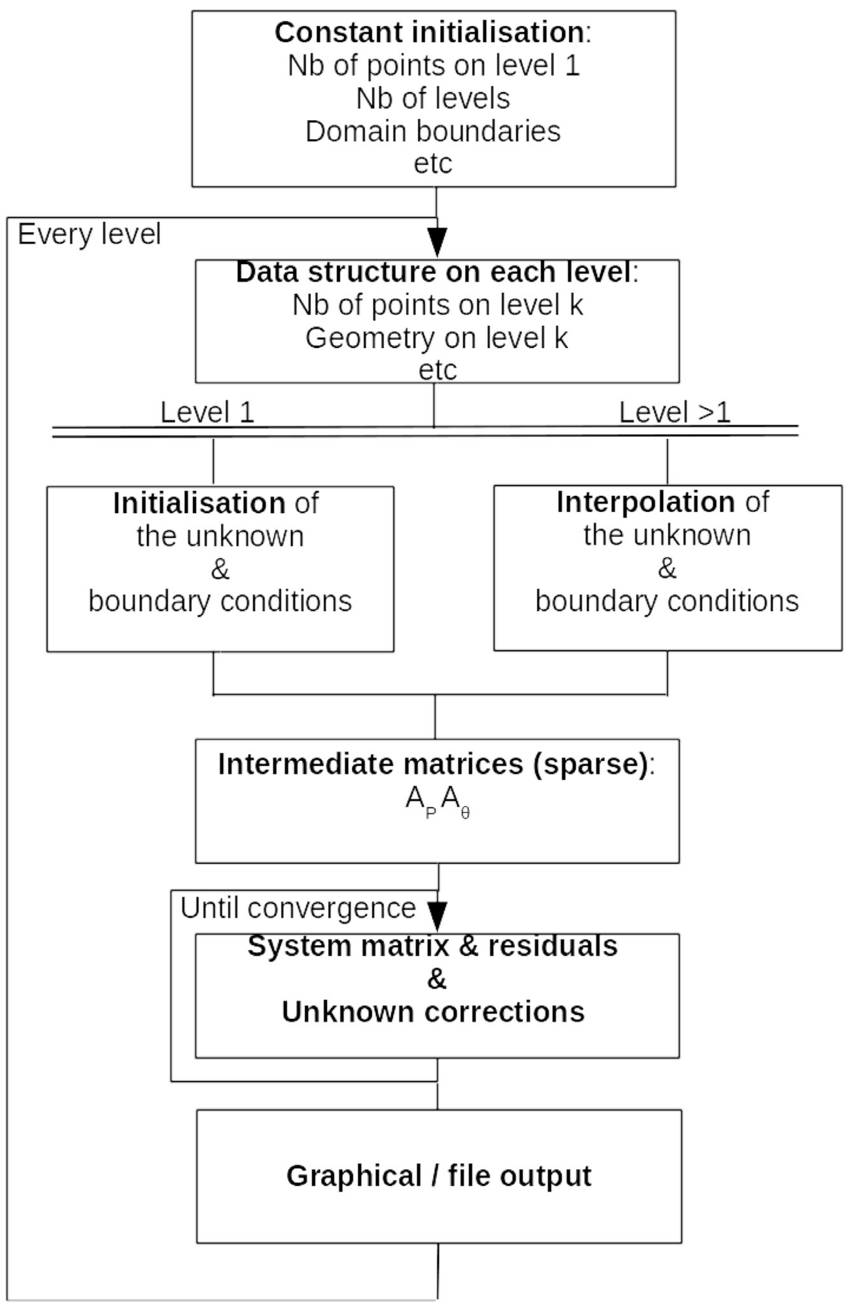

Fig. 2. Algorithm.