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Transient frictionless contact of a rough rigid surface on a viscoelastic half-space

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ABSTRACT

The contact problem of a rough, rigid surface with an elastic or viscoelastic incompressible semi-infinite body is studied in this paper. The problem is solved using a Boundary Element Method coupled with a conjugate gradient method. Viscoelasticity is taken into account with a 'State Variable' approach, making it possible to tackle the transient problem efficiently. The results are compared against Persson's theory of transient viscoelastic contact, showing good agreement in terms of contact area ratio and friction coefficient.

1. Introduction

Predicting the contact surface and friction of rubber materials on a road is of critical interest for the tire industry. This particular contact problem is influenced by the viscoelasticity of rubber and by the multi-scale surface roughness of the road.

There are two main analytical approaches for the contact problem. The first one is a multi-asperity approach, initiated by the pioneering work of Greenwood and Williamson [1]. For a Hertzian contact between a sphere and an elastic half-space analytical solutions exist for both a normal and a tangential (friction) force [2]. The idea of a multi-asperity approach is to generalize these relations by considering an infinite distribution of spherical indentors of different height and different radii. The radius and height distribution are derived from the summit height distribution and the summit curvature distribution of the rough surface under consideration. The approach can be extended to viscoelastic frictional contacts as in [3]. However multi-asperity approaches suffer from a major weakness: they do not account for the influence of the indentors on each other, so they are only valid at small normal loads.

The second approach is the one initiated in [4]. Normal displacements of the half space are supposed to follow the same Power Spectral Density as the rough surface. From this hypothesis the normal contact problem is solved and the friction is deduced in the case of a viscoelastic half space sliding at constant speed. This model is analytically exact for full contact but is less precise as the contact ratio decreases. This defect was modified in [5] to improve results for low contact ratios and has been widely tested by numerical simulations for both elastic [6] and viscoelastic contact [7]. Shortly after his 2001 article, Persson adapted the model in [8] to handle transient sliding. Transient sliding is of interest in tire modeling as the rubber resides in the contact zone for only milliseconds and slides only millimeters. Under such conditions steady state modeling is not accurate. It should be highlighted that in Persson-like approaches the friction is always deduced from the viscoelastic losses in the half space and that no contact friction is taken into account.

A review and a comparison of the two approaches is available in [9].

To overcome the limitations of analytical solutions, numerical simulations are necessary. Numerical simulations are limited by the fact that a very fine mesh is necessary to handle the different length scales of the surface roughness. A 3-dimensional mesh would be excessively big. Using the assumption that the rubber block is very large compared to the size of the simulation and undergoing small strains, simplifies the problem. Using Green's functions, the surface displacements are directly related to the surface stresses, so only the surface needs to be meshed.

Analyzing 2 or 3 length scales of surface roughness still requires more than a million degrees of freedom and consequently long computation times. Typical roads have a surface roughness over more than 6 length scales, which means the simulations should be very fast to take into account as many length scales as possible. Using a multi-level multi-summation technique Brandt and Lubrecht [10] obtain a reduction of the computational cost from $O(N^2)$ to $O(N\log(N))$, N being the number of degrees of freedom (DOF). A number of other techniques have since been developed. Using Fast Fourier Transforms (FFT) as in [11] allows the same reduction provided the mesh is regular and periodic. A slightly slower extension to non-periodic problems is found in [12]. A comparison between multi-level

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and FFT methods is given in [13]. Another approach is to use a mesh smart enough to reduce the number of DOF without impacting precision. This is achieved by refining the mesh at the edges of contact clusters only and keeping a relatively coarse mesh in the inner part. This 'Active Sets' method are developed in [14,15]. Molecular Dynamics solvers were also developed: GFMD [16,17] and RMD [7].

This paper uses Fast Fourier Transforms with a conjugate gradient iteration scheme.

2. Numerical model

Firstly, the contact between an elastic half space and a rigid body without friction is considered. The relation between surface pressure P and surface normal displacement U_z was first found by Boussinesq [18] and is given by:

$$U_z(x,\, y) = \frac{1-\nu}{2\pi G} \iint_{\Omega} \frac{P(\xi,\, \eta)}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} d\xi d\eta \tag{1}$$

G is the shear modulus, ν the Poisson ratio, x, y, ξ and η are spatial variables.

Using the Boussinesq equations Love [19] found the normal displacements for a uniformly distributed pressure over a polygonal region. Discretizing the surface into a uniform mesh with square cells and using these results leads to Eq. (2).

$$U_z(i,\, j) = \frac{1}{G} \sum_{i',j'} A_{zz}(i-i',\, j-j')P(i',\, j') \tag{2}$$

A_{zz} is the Influence Coefficient matrix, which is detailed in [20] and depends on the mesh size. Eq. (2) is a convolution and therefore can be computed very efficiently in Fourier space. Here the problem is considered to be periodic, as periodic random rough surfaces will be used. Using FFT and non-periodic surfaces is also possible with the appropriate zero-padding [12].

Solving the contact problem means finding the pressure in the contact zone satisfying:

$$P \geq 0 U_z - H \geq 0 U_z = H \text{ in the contact zone} P = 0 \text{ outside the contact zone} \tag{3}$$

U_z is the normal displacement calculated from the pressure (Eq. (2)) and H is the rigid substrate height map.

An efficient and easy to implement way to find P is to use an iterative procedure, such as the conjugate gradient method [21,14,15]. For a contact problem the procedure is slightly different from a classical conjugate gradient as the contact surface is not constant. One solution is to solve for a fixed contact surface and change the contact surface by removing the points with negative pressures and including the points where the two surfaces interpenetrate. Then solve again and continue the loop while the contact surface is not constant. This procedure guarantees the existence and uniqueness of the solution and convergence [20]. It is rather slow though, as the Conjugate Gradient has to be executed several times. The usual solution to avoid this is to update the contact surface within the Conjugate Gradient algorithm, between each iteration. This is the solution chosen here.

2.1. Viscoelasticity

Discretizing time into small time steps makes it possible to model the transient response of a viscoelastic half space.

The Zener, or Standard Linear Solid viscoelastic model is used. It can be represented as a spring and a dashpot connected in parallel (elastic shear modulus G_1, viscosity η_1), connected in series to another spring (modulus G_0) - see Fig. 1.

Rubber is considered as an incompressible material. An incompressible viscoelastic material with a Zener law follows the differential Eq. (4), where s and e are the deviatoric parts of the stress and strain

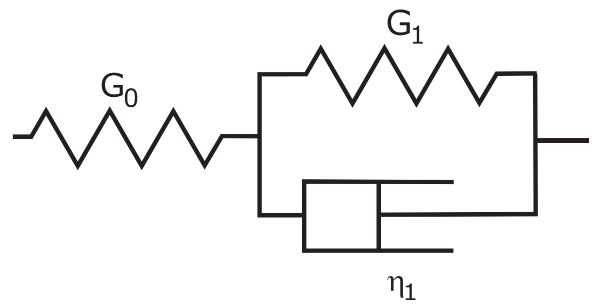

Fig. 1. Representation of the Standard Linear Solid. G_0 and G_1 are the spring stiffness, η_1 the dashpot damping coefficient.

tensors, the dot denotes a time derivative. This differential formulation for viscoelasticity is strictly equivalent to the integral formulation as mentioned in [22].

$$s\left(1 + \frac{G_1}{G_0}\right) + \frac{\eta_1}{G_0}\dot{s} = 2G_1 e + 2\eta_1 \dot{e} \tag{4}$$

Now let us consider a contact problem with a normal pressure imposed on an incompressible viscoelastic half space. This kind of problem can be treated using functional equations [22]. In the Laplace domain, the problem is equivalent to an elastic contact problem, which means it is possible to use analytical elastic solutions, in particular the Boussinesq potential. Going back to the real domain, the viscoelastic Boussinesq equation is given by Eq. (5).

$$\iint_{\Omega} \frac{1}{4\pi\rho}\left(P(\xi,\, \eta)\left(1 + \frac{G_1}{G_0}\right) + \frac{\eta_1}{G_0}\dot{P}(\xi,\, \eta)\right) d\xi d\eta = G_1 U_z(x,\, y) + \eta_1 \dot{U}_z(x,\, y)$$

$$\text{with } \rho = \sqrt{(x-\xi)^2 + (y-\eta)^2} \tag{5}$$

For an implementation in BEM, Eq. (5) is discretized in space and leads to Eq. (6), where i, i', j and j' denote the row and column of the considered mesh cell.

$$\sum_{i',j'} A_{zz}(i-i',\, j-j')P(i',\, j')\left(1 + \frac{G_1}{G_0}\right) + \frac{\eta_1}{G_0} \sum_{i',j'} A_{zz}(i-i',\, j-j')\dot{P}(i',\, j')$$

$$= G_1 U(i,\, j) + \eta_1 \dot{U}(i,\, j) \tag{6}$$

Discretizing time t in the previous equation and removing the sum notations leads to:

$$A_{zz} P_{t+\delta t}\left(1 + \frac{G_1}{G_0}\right) + \frac{\eta_1}{G_0} A_{zz} \frac{\Delta P}{\Delta t} = G_1 U_{t+\delta t} + \eta_1 \frac{\Delta U}{\Delta t} \tag{7}$$

ΔP and ΔU are the variation of pressure and displacement between time step t and time step $t + \Delta t$.

A generalized Zener model is a combination of different Zener models connected in parallel, plus a branch with just one spring G_∞. It is necessary to use it for rubber in order to model its behavior over a large range of frequencies. The displacement in each branch is the same and the pressures in each branch add up. Each branch k follows Eq. (7). It leads to Eq. (8).

$$U_{t+\Delta t}\left[G_\infty + \sum_k \frac{G_1^k + \frac{\eta_1^k}{\Delta t}}{1 + \frac{G_1^k}{G_0^k} + \frac{\eta_1^k}{G_0^k \Delta t}}\right] = A_{zz} \sum_k P_{t+\Delta t}^k + U_t \sum_k \frac{\frac{\eta_1^k}{\Delta t}}{1 + \frac{G_1^k}{G_0^k} + \frac{\eta_1^k}{G_0^k \Delta t}}$$

$$- \sum_k A_{zz} P_t^k \frac{\frac{\eta_1^k}{G_0^k \Delta t}}{1 + \frac{G_1^k}{G_0^k} + \frac{\eta_1^k}{G_0^k \Delta t}} \tag{8}$$

It is then possible to use the 'elastic' Conjugate Gradient contact solver using P', U_z' and H' instead of P, U_z and H according to Eq. (9). This yields $U_{t+\Delta t}$ and $\sum_k P_{t+\Delta t}^k$, which is the total pressure acting on the