European Journal of Mechanics / B Fluids 67 (2018) 25–34

Contents lists available at ScienceDirect

European Journal of Mechanics / B Fluids

journal homepage: www.elsevier.com/locate/ejmflu

The pressure field beneath intense surface water wave groups

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ARTICLE INFO

Article history:
Received 14 March 2017
Received in revised form 1 August 2017
Accepted 1 August 2017
Available online 10 August 2017

Keywords:
Pressure in nonlinear surface waves
Induced pressure
Nonlinear wave groups
Microseismic effect

ABSTRACT

A weakly-nonlinear potential theory is developed for the description of deep penetrating pressure fields caused by single and colliding wave groups of collinear waves due to the second-order nonlinear interactions. The result is applied to the representative case of groups with the sech-shape of envelope solitons in deep water. When solitary groups experience a head-on collision, the induced due to nonlinearity dynamic pressure may have magnitude comparable with the magnitude of the linear solution. It attenuates with depth with characteristic length of the group, which may greatly exceed the individual wave length. In general the picture of the dynamic pressure beneath intense wave groups looks complicated. The qualitative difference in the structure of the induced pressure field for unidirectional and opposite wave trains is emphasized.

1. Introduction

The sea surface motions are accompanied by complicated movements of fluid particles within the entire water column. The underwater fluid dynamics may have significant effect on submerged constructions, formation of bottom topography peculiarities, sediment transport and so on. Obviously, extreme surface waves may cause stronger underwater effects; enormous wave force records at coastal and moored structures are sometimes reported (e.g. [1]) and are often related to so-called rogue wave events (see [2]). Thus, the problem of estimation of underwater extreme pressures and the associated impact on structures has clear practical importance.

On the other hand, subsurface and bottom pressure sensors are commonly used for the registration of surface waves in relatively low-frequency range, from tidal to wind waves. This type of wave recorders is very appropriate as they do not deface the scenery in coastal areas and are difficult to be accessed by unauthorized people (vandal-proof). We may speculate that arrays of submerged probes also could be used for safe and inconspicuous recording of wind waves.

The inverse problem of an accurate recovery of characteristics of wind waves from the data of subsurface/bottom pressure measurements is vital. The pressure under very long waves such as tides and tsunamis may be assumed hydrostatic; this approximation is sufficient for an accurate reconstruction of the surface waves from the bottom pressure sensor data, what enables creating of tsunami early warning networks. However, wind waves are often not sufficiently long even in the coastal area. Then nonhydrostatic corrections to the pressure contribute significantly and the relation between the surface displacement and the pressure at the location of registration becomes complicated. As the inverse problem is highly attractive, it has motivated a number of studies. The linear theory based on the spectral methods was developed in [3–10], though the importance of nonlinear effects was demonstrated shortly after. Most of the studies concern shallow water conditions; it is reasonable that representative solutions such as uniform waves, shallow water solitons and their interactions are thoroughly studied [11–19], and model theories are being validated against the direct solution of hydrodynamic equations.

The solutions of the Laplace equation for potential planar surface waves inevitable possess the feature that each surface wave harmonic with a given horizontal scale attenuates with depth with exactly the same vertical scale. Thus, short waves have no effect on sufficiently deep water layers. This picture gets broken due to the nonlinear wave interactions which may lead to the generation of longer wave components, which penetrate much deeper into the water bulk. This mechanism was suggested by Longuet-Higgins [20] to explain the seismic noise generated by wind waves, which would not produce noticeable effect on the sea bottom over relatively deep areas in the linear regime. Longuet-Higgins [20] suggested a simple solution for two counter propagating uniform waves (i.e., standing waves), which due to the nonlinear interaction

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http://dx.doi.org/10.1016/j.euromechflu.2017.08.002

produce a harmonic with the frequency $\omega_0 + \omega_0 = 2\omega_0$ and the wavenumber $k_0 - k_0 = 0$. As a result, the second-order pressure does not attenuate at large depth at all. A more general treatment of the problem when the waves obey some spectrum was given by Hasselmann [21]. The approach by Longuet-Higgins [20] seems to prevail in the literature probably due to its simplicity. A review of the nonlinear sea wave interaction theory in the context of a seismic wave generation may be found in monograph by Kibblewhite & Wu [22]. The effect of sea surface waves on a seismic noise is usually considered in the statistical sense, when the generated seismo-acoustic spectra are discussed.

In this paper we consider the underwater pressure fields beneath isolated intense wave groups. The groups own a new characteristic length scale, which is typically O (10) times larger than the wave length. Due to the nonlinear wave interactions the modulated waves generate large-scale perturbations which decay with depth with the typical scale of the group length [23–26]. In that way, a given water basin in terms of group lengths is not as deep as in terms of individual wave lengths.

The group structure of sea waves is specified not solely by the given wave spectrum, but also by the dynamical effects of the modulations. In particular, in deep water regions intense waves tend to split into the wave groups characterized by an approximate balance of dispersive and nonlinear effects (i.e., envelope solitons, nonlinear solitary groups, [27,28]). The experimental observations of intense solitary groups are reported in [29,30]. The balance between the nonlinearity and dispersion allows the groups to approximately preserve the shape for long time. Therefore, besides regular waves and shallow water solitons, the nonlinear solitary groups are a representative wave structure in the sea. The pressure induced by these groups is in focus of this study.

Our approach is not limited by the simplified formulation suggested by Longuet-Higgins [20]. The wave dispersion is taken into account by exact solution of the Laplace equation for potential waves. The surface boundary conditions which correspond to the nonlinearly generated long perturbations are obtained with the help of the weakly nonlinear asymptotic approach for modulated wave trains. The focus on narrow frequency spectra allows us to simplify the approach by Hasselmann [21] and to obtain eventually closed form expressions for the solution which may be straightforwardly solved numerically with the help of the Fourier transform subroutine. For the case of wave groups having the shape of the envelope solitons of the nonlinear Schrödinger equation, analytic solutions in terms of special functions are obtained.

The paper is organized as follows. The classic framework of potential planar water waves is given briefly in Section 2. In particular the solution of the Laplace equation with the help of the Fourier method is outlined. The asymptotic theories for the large-scale perturbations induced by unidirectional and counter propagating modulated waves are given in Section 3. They determine the surface boundary conditions for the Laplace equation. The details of the derivation of the theory for opposite waves are collected in Appendix A. The resulting formulas for the underwater dynamic pressure are given in Section 3. They require the knowledge of the space–time evolution of the squared wave envelope. The particular case of the wave groups having sech-shapes of the nonlinear Schrödinger equation envelope solitons is discussed in Section 4 in details.

2. The pressure field in the volume of potential surface waves

We confine the consideration to the framework of the two-dimensional potential Euler equations for ideal incompressible inviscid fluid. Then the motions of the fluid in the water volume from the surface $\eta(x,t)$ to the flat bottom $z = -h$ are governed by the Laplace equation on the velocity potential $\varphi(x, z, t)$,

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial z^2} = 0, \quad -h \le z \le \eta(x, t). \tag{1}$$

The boundary condition at the bottom requires a zero vertical component of the velocity,

$$\frac{\partial\varphi}{\partial z} = 0 \quad \text{at } z = -h, \tag{2}$$

which in the infinitely deep water limit, $h \to \infty$, transforms to the condition $\varphi \to 0$. The surface boundary condition is nonlinear and consists of the dynamical and kinematic conditions respectively,

$$g\eta + \frac{\partial\varphi}{\partial t} + \frac{1}{2}\left(\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2\right) = 0 \quad \text{at } z = \eta, \tag{3}$$

$$\frac{\partial\varphi}{\partial z} = \frac{\partial\eta}{\partial t} + \frac{\partial\eta}{\partial x}\frac{\partial\varphi}{\partial x} \quad \text{at } z = \eta, \tag{4}$$

where g is the acceleration due to gravity.

If the velocity potential at the rest water level is specified, $\Phi(x, t) \equiv \varphi(x, z = 0, t)$, then the potential in the entire volume occupied by water may be straightforwardly obtained with the help of the Fourier method,

$$\varphi(x, z, t) = \hat{F}^{-1}\left\{\hat{\Phi}(\omega, k)\frac{\cosh(k(z+h))}{\cosh(kh)}\right\} \tag{5}$$

where the double Fourier direct and inverse transforms are defined in the form

$$\hat{F}\{r(x, t)\} \equiv \frac{1}{(2\pi)^2}\iint r(x, t)\, e^{-i\omega t + ikx} dx dt = \hat{r}(\omega, k),$$

$$\hat{F}^{-1}\{\hat{r}(\omega, k)\} = \iint \hat{r}(\omega, k)\, e^{i\omega t - ikx} d\omega dk = r(x, t). \tag{6}$$

In (6) the integration is assumed to perform over infinite domains in time and space. One may note that the Laplace equation does not contain time dependence, thus the solution inherits it from the surface velocity potential. Also, the double Fourier transformation may be replaced by the transformation in either time or space if the relation between the wavenumbers and frequencies is given.

The present study is focused on relatively large depth. In the limit of very deep water the vertical structure of the modes may be simplified, and then formula (5) transforms to

$$\varphi(x, z, t) = \hat{F}^{-1}\left\{\hat{\Phi}(\omega, k)\, e^{|k|z}\right\}. \tag{7}$$

When the pressure at the surface is assumed to be zero, the total pressure in the water volume P_{tot} is defined by the Bernoulli law

$$\frac{1}{\rho}P_{tot} = -\frac{\partial\varphi}{\partial t} - \frac{1}{2}\left(\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2\right) - gz, \tag{8}$$

where the constant ρ is the water density ($P_{tot} = 0$ at $z = \eta$ according to (3)). The normalized dynamic pressure which characterizes the excess of the total pressure over the hydrostatic pressure is given by

$$p \equiv \frac{1}{\rho}P_{tot} + gz = -\frac{\partial\varphi}{\partial t} - \frac{1}{2}\left(\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2\right). \tag{9}$$

The two last terms in (9) are nonlinear and may be neglected compared to the time derivative of φ, if the wave amplitude is small. In what follows the dynamic pressure will be approximated by the leading-order part of (9), i.e.

$$p \approx -\frac{\partial\varphi}{\partial t}. \tag{10}$$