European Journal of Mechanics / B Fluids 67 (2018) 65–69

Contents lists available at ScienceDirect

European Journal of Mechanics / B Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Far-wake flows downstream of cylinders: A novel generalized similarity method

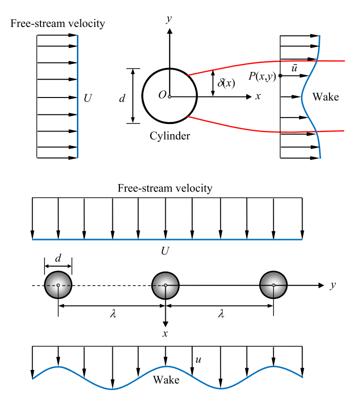
Sujit K. Bose [a], Subhasish Dey [b,c,d,*]

[a] Centre for Theoretical Studies, Indian Institute of Technology Kharagpur, West Bengal 721302, India
[b] Department of Civil Engineering, Indian Institute of Technology Kharagpur, West Bengal 721302, India
[c] Physics and Applied Mathematics Unit, Indian Statistical Institute, Kolkata, West Bengal 700108, India
[d] Department of Hydraulic Engineering, State Key Laboratory of Hydro-Science and Engineering, Tsinghua University, Beijing 100084, People's Republic of China

HIGHLIGHTS

- Turbulent far-wake flow downstream of a cylinder is analyzed by a generalized similarity method.
- This method is then applied to the problem of an array of cylinders.
- Both the turbulent and laminar far-wake flow cases are analyzed for the latter problem.

GRAPHICAL ABSTRACT



ARTICLE INFO

Article history:
Received 5 March 2017
Received in revised form 12 July 2017
Accepted 9 August 2017
Available online 14 August 2017

Keywords:
2-D wake
Clarkson–Kruskal similarity method
Fluid dynamics
Fluid flow
Hydraulics
Hydrodynamics
Oseen-type approximation
RANS
Turbulent flow
Wakes

ABSTRACT

The nonlinear momentum equation for turbulent far-wake flow downstream of a cylinder is derived from the Reynolds averaged Navier–Stokes equations and solved by iteration and generalized similarity solution. The first-order Oseen-type linearization yields for the wake boundary layer width a series, whose terms vary as $x^{1/2}$, $x^{-1/2}$, $x^{-3/2}$ etc., where x is the streamwise distance from the center of cylinder. The velocity deficit from the free-stream velocity is similarly obtained as a series and the iteration is carried over to a second-order. This method is then applied to the problem of an array of cylinders. Both turbulent and laminar far-wake flow cases are analyzed. The series solutions broadly extend the earlier findings. The first-order solutions agree in form with those stated in the book of Schlichting (1979).

1. Introduction

Theoretical study of two-dimensional far-wake flow downstream of cylindrical bluff-bodies has attracted much attention of

* Corresponding author at: Department of Civil Engineering, Indian Institute of Technology Kharagpur, West Bengal 721302, India.
E-mail addresses: sdey@iitkgp.ac.in, sdey@civil.iitkgp.ernet.in (S. Dey).

http://dx.doi.org/10.1016/j.euromechflu.2017.08.008
0997-7546/

Nomenclature

$A, B, C_{0\text{-}4}, D, K$	constants
C_D	drag coefficient
d	diameter of cylinder
$\overline{p}$	time-averaged hydrostatic pressure
U	free-stream velocity
u, v	instantaneous streamwise and transverse velocity components, respectively
$\overline{u}, \overline{v}$	time-averaged values of u and v, respectively
u', v'	fluctuations of u and v, respectively
u_0	peak velocity deficit
$\overline{u}_1$	time-averaged velocity deficit
x	longitudinal distance
y	transverse distance
α	exponent
δ	half-width of wake
ε	eddy viscosity
ε_0	constant
ϕ	function of velocity deficit
η	y/δ
λ	spacing
ψ	function of Reynolds shear stress
υ	coefficient of kinematic viscosity of fluid
σ	pressure plus normal Reynolds stress
ξ	$x^{1/\alpha}$

several authors due to varied analytical treatments of the general wake flow phenomenon. In the far-wake flow region, both laminar and turbulent flow cases are succinctly discussed by Schlichting [1]. In zero-pressure gradient laminar flow, under the first-order *Oseen-type* approximation of the nonlinear Navier–Stokes equations, called the *Prandtl equations*, it was shown by order of magnitude analysis that the half-width δ of the wake boundary layer varies as $x^{1/2}$ and the velocity deficit $\overline{u}_1$ varies as $x^{-1/2}$. Here, x is the streamwise distance (along the free-stream) from the center of cylinder, $\overline{u}_1 = U - \overline{u}$, U is the free-stream velocity and $\overline{u}$ is the time-averaged velocity in x-direction in the wake domain. In these theories, the laws of variations of δ and $\overline{u}_1$ with x hold for both laminar and turbulent flow cases. The variation of $\overline{u}_1$ with the transverse distance y is approximately of the Gaussian form. Abandoning the concept of the Oseen-type linearization, the nonlinear laminar far-wake flow was analyzed by several authors [2–8]. The study of the turbulent flow case by George [9] was however based on basic similarity considerations. In addition, studies on laminar flow structures around circular cylinders were conducted by Alfonsi et al. [10,11].

In this study, the solutions for the far-wake flows downstream of cylinders are obtained by using the generalized Clarkson–Kruskal similarity method [12], as in the case of a circular far-wake flow downstream of a sphere [13]. Both laminar and turbulent flow cases are treated. The governing nonlinear momentum equation in turbulent flow case is derived from the Reynolds averaged Navier–Stokes (RANS) equations. In the first-order Oseen-type approximation, the self-similarity of the equation requires the pressure gradient to be zero and the half-width δ of wake to satisfy a nonlinear differential equation. The equation of δ is solved in a general series form. This solution is then adopted for two-dimensional wake flow problem and the solution for the velocity deficit $\overline{u}_1$ is obtained. The variation of velocity deficit $\overline{u}_1$ in y-direction has a Gaussian form. As in Schlichting [1], the flow past an array of cylinders is then analyzed by using the present method. Finally, the laminar wake flow problem is also solved. In the last two cases, the first-order solutions agree with those stated in Schlichting [1].

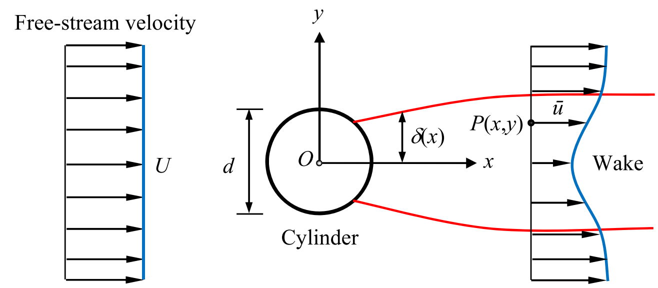


Fig. 1. Schematic of two-dimensional wake flow downstream of a circular cylinder.

2. Momentum equation of wake flow

Two-dimensional wake flow past a circular cylinder is shown schematically in Fig. 1. The free-stream flow velocity is assumed to be U emerging from $x \to -\infty$. In turbulent wake flow region, at any point $P(x, y)$, is governed by the continuity equation and the RANS equation in x-direction:

$$\overline{u}_x + \overline{v}_y = 0 \tag{1}$$

$$\overline{u} \cdot \overline{u}_x + \overline{v} \cdot \overline{u}_y = -\overline{p}_x - \left(\overline{u'v'}\right)_y + \upsilon \overline{u}_{yy} - \left(\overline{u'u'}\right)_x \tag{2}$$

where $(\overline{u}, \overline{v})$ and (u', v') are the time-averaged and fluctuations of instantaneous velocity components (u, v) at the point (x, y), respectively, $\overline{p}$ is the time-averaged hydrostatic pressure normalized by mass density of fluid and υ is the coefficient of kinematic viscosity of fluid. The subscripts refer to partial derivatives. In the wake flow region, the $\overline{u}$ is less than U and the $\overline{u}_1$ is small as compared to U. Using Eq. (1), Eq. (2) yields the nonlinear momentum equation as

$$-U(\overline{u}_1)_x + \overline{p}_x + \left(\overline{u'v'}\right)_y + \upsilon(\overline{u}_1)_{yy} + \left(\overline{u'u'}\right)_x = -\overline{u}_1(\overline{u}_1)_x + (\overline{u}_1)_y \int_0^y (\overline{u}_1)_x dy \tag{3}$$

To a first-order of $\overline{u}_1$, the above equation yields the Oseen-type approximate one-dimensional momentum equation:

$$-U(\overline{u}_1)_x + \overline{p}_x + \left(\overline{u'v'}\right)_y + \left(\overline{u'u'}\right)_x = -\upsilon(\overline{u}_1)_{yy} \tag{4}$$

Let $y = \pm\delta(x)$ be the equation of the wake boundary layer (Fig. 1). Experiments by Schlichting [14] and Reichardt [15] revealed that the flow is self-similar beyond some streamwise distance from the cylinder. For the solution of Eq. (4), a generalized Clarkson–Kruskal self-similar solution is sought in terms of the nondimensional variable $\eta = y/\delta(x)$ in the following forms [12]:

$$\overline{u}_1 = u_0(x)\,\phi(\eta)\,, -\overline{u'v'} = [u_0(x)]^2 \psi(\eta)\,, \left(\overline{p} + \overline{u'u'}\right)_x = [u_0(x)]^2 \sigma(\eta) \tag{5}$$

where u_0 is the peak velocity deficit and ϕ, ψ and σ are the functions of velocity deficit, Reynolds shear stress and pressure plus normal Reynolds stress, respectively.

Substitution of Eq. (5) into Eq. (4) and multiplication by δ/u_0^2, results in following:

$$-U\left(\frac{\delta}{u_0^2} \cdot \frac{du_0}{dx}\phi - \frac{\eta}{u_0} \cdot \frac{d\delta}{dx}\phi'\right) - \psi' + \delta\sigma = -\frac{\upsilon}{u_0\delta}\phi'' \tag{6}$$

where prime and double prime denote first and second derivative of the functions, respectively. The coefficient $\upsilon/(u_0\delta)$ containing kinematic viscosity on the right hand side of Eq. (6) is negligible due to turbulent flow case. Further, for a self-similar solution to exist, σ must vanish, as $\delta \neq$ constant. It implies that $\left(\overline{p} + \overline{u'u'}\right)_x$