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On the use of linear theory to estimate bottom pressure distribution under nonlinear surface waves

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HIGHLIGHTS

- Travelling solitonic waves, and transient wave packets are propagated numerically using two distinct approaches.
- The first one is based on fully nonlinear potential equations, while the second one is based on the Green–Naghdi system of equations.
- Bottom pressure distributions are compared to both space and time linear theory.
- Qualitative and quantitative behaviour are compared. The role of dispersion in the physical process is emphasized.

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ABSTRACT

The bottom pressure distribution beneath large amplitude waves is studied within linear theory in time and space domain, weakly dispersive Serre–Green–Naghdi system and fully nonlinear potential equations. These approaches are used to compare pressure fields induced by solitary waves, but also by transient wave groups. It is shown that linear analysis in time domain is in good agreement with Serre–Green–Naghdi predictions for solitary waves with highest amplitude $A = 0.7h$, h being water depth. In the meantime, when comparing results to fully nonlinear potential equations, neither linear theory in time domain, nor in space domain, provide a good description of the pressure peak. The linear theory in time domain underestimates the peak by an amount similar to the overestimation by linear theory in space domain. For transient wave groups (up to $A = 0.52h$), linear analysis in time domain provides results very similar to those based on the Serre–Green–Naghdi system. In the meantime, linear theory in space domain provides a surprisingly good comparison with prediction of fully nonlinear theory. In all cases, it has to be emphasized that a discrepancy between linear theory in space domain and in time domain was always found, and presented an averaged value of 20%. Since linear theory is often used by coastal engineers to reconstruct water elevation from bottom mounted sensors, the so-called inverse problem, an important result of this work is that special caution should be given when doing so. The method might surprisingly work with strongly nonlinear waves, but is highly sensitive to the imbalance between nonlinearity and dispersion. In most cases, linear theory, in both time and space domain, will lead to important errors when solving this inverse problem.

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1. Introduction

For various experimental reasons, the measurement of water waves propagating in shallow water environments such as surf zones or coastal areas is a difficult task. These measurements are often performed using bottom mounted pressure sensors. The data collected might be inverted to provide the related water elevation. However, the function used to perform this inversion is subject to question. Indeed, when considering very long waves, like tides and tsunamis, the pressure is hydrostatic as long as dispersive effects can be neglected, and recovering surface elevation from the bottom pressure does not imply any particular difficulty.

On the other hand, the propagation of water waves in coastal areas are more complex. In such areas, one may find wind waves, which are not long waves even in the coastal zone. The corrections related to their dispersive behaviour might play a significant role. The wave behaviour near the coast (cliffs or vertical barriers) in the process of the wave reflection should also be taken into account.

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For instance, the relation between wave elevation, and bottom pressure is not straight forward, due to the interaction of incident and reflected wave [1,2].

Anyway, spectral methods based on transfer functions are often used to reconstruct the water elevation taking into account the assumption of linearity of waves [3–10]. The reasons why this assumption might be questioned are twofold. First, when considering the transfer function between elevation and pressure, nonlinear terms are obviously neglected within the linear assumption. Secondly, the linear dispersion relation does not take nonlinear dispersion into account. In shallow water environment, nonlinear dispersion becomes predominant with respect to linear dispersive effects. Beyond these two reasons, when considering the inverse problem, i.e. reconstructing water waves elevation starting from pressure records, the problem is ill-posed, and further difficulties appear.

Since the work of Bishop and Donelan [5], the linear hypothesis is often considered not to hold for large amplitude waves. Indeed, these authors found the linear prediction to underestimate the elevations of about 15% for largest waves. Thus, several authors found an expression when considering nonlinear periodic waves or solitary waves [11–14]. The pressure map beneath these waves might be found in [15,16,17]. However, since the possible errors involved in the process are twofold, it might be possible that they compensate each other, providing a better correspondence than expected. Indeed, depending on how nonlinear dispersive effect compare with frequency dispersion effect, nonlinear terms involved within the transfer function might be compensated by nonlinear error in dispersion relation.

To our knowledge, the combined effect of these two mechanism has not been discussed yet. The main purpose of this work is to investigate the ability of linear theory to correlate surface elevation with bottom pressure distribution. To achieve this goal, two cases are considered. First, a strongly nonlinear solitary wave is considered. Then, a strongly nonlinear transient wave group is studied. This approach allows to vary the respective role of frequency and nonlinear dispersion. In the first case, linear and nonlinear dispersive effects are of same order. In the second case, the frequency dispersion is predominant.

These two reference cases are propagated using alternatively the fully nonlinear potential equations, and the weakly dispersive, fully nonlinear Serre–Green–Naghdi system. The bottom pressure evolution is obtained numerically in both cases. The bottom pressure obtained in the framework of the fully nonlinear potential equations serves as reference, while the distribution obtained in the framework of Serre–Green–Naghdi system provides a fully nonlinear weakly dispersive solution.

In all cases, bottom pressure distribution is computed using the linear theory in both time and space domain, and compared with the distributions obtained numerically. This approach provides new insight in interpreting the respective roles of linear dispersion, nonlinear dispersion, and nonlinear terms involved in the transfer function.

Numerical models used in this work are presented in Section 2 (fully nonlinear potential equations), and in Section 3 (Serre–Green–Naghdi system). Results of computations for travelling solitary waves and transient wave groups discussed respectively in Sections 5.1 and 5.2. In these sections, the respective roles of various terms involved in linear transfer functions to correlate surface elevation with bottom pressure are emphasized. It is generally admitted that linear theory in time domain should not be used to reconstruct elevation from bottom pressure distribution [5,16] when wave fields are nonlinear, and this result finds a new confirmation here. In the meantime, conclusions about linear theory in space domain are not straightforward. These new results are discussed in Section 6.

2. Numerical solution of the fully nonlinear equations

2.1. Basic equations of the problem

As it is classically done, the fluid is assumed to be inviscid and incompressible. The further hypothesis of irrotational motion allows the velocity to derive from a velocity potential, $u = \nabla\phi$, where the velocity potential $\phi(x, z, t)$ has to be solution of Laplace's equation. Here, x and z refer respectively to the horizontal and vertical space coordinates, whereas t denotes time. The coordinate $z = 0$ corresponds the location of the free surface at rest, while an horizontal impermeable bed is located at $z = -h$. Finally, the numerical domain is closed at its two remaining extremities $x = 0$ and $x = L$ by vertical impermeable walls.

Accordingly to the dynamic free surface condition, the pressure at $z = \eta(x, t)$ has to be nil. Together with the kinematic free surface condition, which expresses the impermeability of the free surface, and the bottom condition, Laplace's equation might be solve.

$$\begin{cases} \Delta\phi = 0 & \text{in } -h \le z \le \eta(x,t), \\ \dfrac{\partial\eta}{\partial t} + \dfrac{\partial\phi}{\partial x}\dfrac{\partial\eta}{\partial x} = \dfrac{\partial\phi}{\partial z} & \text{on } z = \eta(x,t), \\ \dfrac{\partial\phi}{\partial t} + \dfrac{(\nabla\phi)^2}{2} + g\eta = 0 & \text{on } z = \eta(x,t), \\ \dfrac{\partial\phi}{\partial z} = 0 & \text{on } z = -h. \end{cases} \tag{1}$$

Here, g is the acceleration due to gravity, and ρ the water density. Once the velocity potential and its gradient are known in the fluid, the bottom pressure is obtained by using Bernoulli's equation

$$\frac{p}{\rho} = gh - \frac{\partial\phi}{\partial t} - \frac{(\nabla\phi)^2}{2} \quad \text{on} \quad z = -h. \tag{2}$$

2.2. Numerical approach

The system of Eqs. (1) is solved using a classical Boundary Integral Equation Method (BIEM). The free surface is treated with a mixed Euler Lagrange (MEL) time marching scheme. The numerical approach used here is fully documented in [18] for general cases, and its use to investigate the propagation of solitonic waves can be found in [19,2,20].

The method is based on the use of Green's third identity, to solve Laplace's equation for the velocity potential.

$$\int_{\partial\Omega} \Phi(\mathrm{P}) \quad \frac{\partial G}{\partial n}(\mathrm{P},\mathrm{Q})d\ell - \int_{\partial\Omega} \frac{\partial\Phi}{\partial n}(\mathrm{P})G(\mathrm{P},\mathrm{Q})d\ell = c(\mathrm{Q})\Phi(\mathrm{Q}), \tag{3}$$

where G is the free space Green's function. The fluid domain boundary $\partial\Omega$ is $\partial\Omega_B \cup \partial\Omega_F$, which correspond respectively to solid boundaries and to the free surface boundary. Since P and Q refer to two points of the fluid domain, and since $c(\mathrm{Q})$ is given by

$$c(\mathrm{Q}) = \begin{cases} 0 & \text{if Q is outside the fluid domain } \Omega \\ \alpha & \text{if Q is on the fluid boundary } \partial\Omega \\ 2\pi & \text{if Q is inside the fluid } \Omega, \end{cases} \tag{4}$$

α being the inner angle delimited by the adjacent panels of the boundary, a discretization of this integral equation can be obtained. Time stepping is performed by means of a fourth order Runge & Kutta scheme, with a constant time step. The bottom pressure is calculated by using a finite-difference method.

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