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Turbulent energy production peak and its location from inner most log law or power law velocity in a turbulent channel/pipe and Couette flows

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ABSTRACT

The present work deals with the inner most, log law velocity and inner most power law velocity, and the associated Reynolds shear stresses, for turbulent energy production in the buffer layer of a fully developed turbulent channel or pipe and Couette flow. The Reynolds momentum equations have been are analyzed with out any closure model of eddy viscosity, mixing length etc. The equivalence of power law solutions with log law solution is demonstrated for large Reynolds numbers. Turbulent energy production asymptotic theory is presented. In a fully developed turbulent channel/pipe flow the theory shows that the peak of production and its location are universal numbers for large friction Reynolds numbers, but for lower Reynolds number theory show dependence on inverse of friction Reynolds number R_τ^{-1}. For turbulent Couette flow peak of production and its location are universal numbers for all friction Reynolds numbers. The turbulent energy production theory predictions in the buffer layer are compared with experimental and DNS data which support the predictions, that in channel or pipe the prediction depend on friction Reynolds number dependence like R_τ^{-1} and for Couette flow the predictions are universal numbers.

1. Introduction

The velocity and Reynolds shear stress distributions in fully developed turbulent flow in a pipe or channel and Couette flow have been of great interest. The equations of turbulent motion are not closed unless a turbulence closure model is adopted. The approaches adopted by Izakson [1], Millikan [2] and Kolmogorov [3] are model free and appeal to an overlap hypothesis. Izakson–Millikan–Kolmogorov (IMK) Hypothesis [4,5] is stated as: Between the viscous and the energetic scales in any turbulent flow exists an overlap domain over which the solutions characterizing the flow in the two corresponding limits must match as Reynolds number tends to infinity. The resemblance of IMK Hypothesis with conventional matching associated with closed equation seem peculiar to turbulence theory.

The overall description of turbulent shear flow has been in terms of two separate length scales (inner wall and outer wake) at large Reynolds numbers. The log law velocity distributions were considered [4–7]. Afzal [6] for turbulent Couette flow, analyzed the turbulent kinetic energy of fluctuations based on Reynolds open equations at large Reynolds number and in the overlap region proposed expressions on page 169 for turbulent energy dissipation Eq. (42), turbulent kinetic energy of velocity fluctuations Eq. (43) and turbulent energy diffusion Eq. (44). The predictions of Afzal [6], based on Reynolds equations with out any closure model (like eddy viscosity, mixing length etc.) can be used for turbulent channel and pipe flows where functional form of predictions remains same but constant may changed they may be estimated from data of pipe/channel flow. The power law velocity distributions were proposed [8–11] in the same overlap region of Izakson and Millikan. For asymptotic large Reynolds number $R_\tau \to \infty$ the log laws and power laws velocity profiles are equivalent solutions.

Ramis, Fransson and Alfredsson [12] observed that the overlap region of Izakson and Milikan is still far from being established in DNS, so that the debate, logarithmic versus power law as well as the irrespective coefficients or constants, is for the time being dependent on quality experiments in canonical wall-bounded flows. In fact observed differences between the different flows within the buffer region are unlikely to be Reynolds number effects while comparing the channel flow DNS data. The DNS data bases used in the present paper, albeit of low Reynolds number, indicate no

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conclusive Reynolds number trends. For $Re_\tau > 500$, the peak position resides with in $y_+ = 14$ and $y_+ = 15$ wall units. It has been emphasized that the necessity of an accurate determination of the wall position in near wall measurements of buffer layer in wall bounded turbulent flows. When not accounted for wall position properly, the data can lead to wrong conclusions about the wall position and there by also on the near-wall behavior of mean and turbulence quantities. While the apparent differences are small and negligible in regards of their effect on the extracted friction velocity, it will be shown later on that the determined wall position is dependent on the chosen relation and the log law constants. This becomes especially important when theme assured velocity profiles do not reach into the viscous sublayer or even buffer region as is the case for most of the high Reynolds number experiments.

The wall bounded turbulent shear flow, has been divided into the viscous sublayer region $0 \le y_+ < 5$, buffer zone region $5 \le y_+ < 30$ and turbulent core $y_+ > 30$, and these numerical constants may slight change from data to data. The buffer layer acts as a cushion between the sublayer and the main flow where turbulence is fully developed flow. In buffer zone, another log law velocity profile was proposed by Karman [13] and Afzal [14,15] where constants κ_i and B_i in buffer layer are different form traditional Izakson–Millikan overlap region constants κ and B. The intermediate layer (mesolayer of Afzal [14,15]) associated with the peak of Reynolds shear stress domain $y \sim O(\nu\delta/u_\tau)^{1/2}$ proposed existence of two overlap regions. In first overlap region the mesolayer is matched with outer layer leading to traditional log region [1,2] in the domain $O(\nu\delta/u_\tau)^{1/2} < y < O(\delta)$ and in second overlap region the mesolayer is matched with inner layer, leading to another log region [14,15] in the domain $O(\nu/u_\tau) < y < O(\nu\delta/u_\tau)^{1/2}$.

The aim of this note is study another overlap region (in the buffer layer) as a part of the generalized log law velocity and power law velocity for predicting the peak of turbulent energy production in fully developed turbulent channel or pipe and Couette flows. It is proposed that in the buffer layer the power law velocity is $u_+ = C_i\, y_+^{\alpha_i}$ with power index α_i and prefactor C_i. It is shown that power index $\alpha_i \to 0$ and prefactor $C_i \to \infty$ such that product $\alpha_i C_i$ is order unity, and for large Reynolds numbers the envelope of power law velocity approaches to log law velocity The turbulent energy production in turbulent channel/pipe and Couette flow have been considered both for log law and power law in the buffer layer domain.

2. Turbulent flow in a channel or pipe:

The Reynolds mean momentum equation, in fully developed turbulent flow in a channel or pipe is

$$\frac{du_+}{dy_+} + \tau_+ = 1 - R_\tau^{-1} y_+ \tag{1}$$

The boundary conditions on the wall and axis of channel or pipe $y = \delta$ are

$$y_+ = 0, \quad u_+ = \tau_+ = 0, \qquad y = \delta, \qquad u = U_c, \quad \frac{du_+}{dy_+} = \tau_+ = 0 \tag{2}$$

Here $u_+ = u/u_\tau$ and $\tau_+ = \tau/\rho u_\tau^2$ are non-dimensional velocity and Reynolds shear stress. u_τ is the friction velocity and U_c is the center line velocity in channel/pipe flow at $y = \delta$. The friction Reynolds number is $R_\tau = u_\tau\delta/\nu$ and traditional Reynolds number $Re = 2\delta U_b/\nu$ is based on U_b the bulk average velocity in a channel or pipe. In the sublayer (immediate neighborhood of the wall) the velocity u_+, Reynolds shear stress τ_+ and turbulent energy production $P_+ = \tau_+\partial u_+/\partial y_+$ for large Reynolds numbers are

$$u_+ = y_+ - \frac{b}{4} y_+^4, \qquad \tau_+ = b\, y_+^3, \qquad P_+ = b y_+^3 \tag{3a, b, c}$$

Guo [7] proposed $b = 1/1150 = 8.696 \times 10^{-4}$ as constant. Ramis, Fransson and Alfredsson [12] proposed that b depends on Reynolds number and in their Table 1 (page 36) computed $1.7 \le b \times 10^4 \le 7.89$ in the Reynolds number range $64 \le R_\tau \le 2000$. The second order correction b to the linear law of the wall, asymptotes to zero with increasing Reynolds number, and becomes practically negligible already for $Re_\tau > 300$.

The turbulent energy production $P_+ = \tau_+ du_+/dy_+$ from Reynolds mean momentum equation (1) for a channel or pipe may be expressed as

$$P_+ = \tau_+\frac{du_+}{dy_+} = (1 - R_\tau^{-1} y_+)\frac{\partial u_+}{\partial y_+} - \left(\frac{\partial u_+}{\partial y_+}\right)^2 \tag{4}$$

In traditional picture the turbulent flow consists of inner wall layer velocity profile $u/u_\tau = u_+(y_+)$ in wall variable $y_+ = y u_\tau/\nu$ and outer layer velocity profile is $(u - U_c)/u_\tau = U_o(Y)$ in outer variable $Y = y/\delta$ for large friction Reynolds number, $R_\tau = u_\tau\delta/\nu \to \infty$. In the overlap region, Izakson [1] proposed a functional equation of velocity, while Millikan [2] considered its differential form and Kolmogorov [3] considered velocity fluctuations. The matching of velocity in inner and outer layers by Izakson–Millikan–Kolmogorov (IMK) Hypothesis [4,5] leads to a functional equation

$$u_+(y_+) = U_{c+}(R_\tau) + U_o(Y), \qquad U_{c+}(R_\tau) = \frac{U_c}{u_\tau} \tag{5}$$

The first differential of the functional equation (5) becomes

$$y_+\frac{du_+}{dy_+} = Y\frac{dU_o}{dY} = R_\tau\frac{dU_{c+}}{dR_\tau} = J \tag{6}$$

In order to further explore, the functional equations. (5) and (6), it is differentiated once more with respect to y, to get

$$y_+^2\frac{\partial^2 u_+}{\partial y_+^2} = Y^2\frac{\partial^2 U_o}{\partial Y^2} = R_\tau^2\frac{d^2 U_{c+}}{dR_\tau^2} = K \tag{7}$$

The turbulent energy production P_+ Eq. (4), in the light of relation (6) becomes

$$P_+ = (1 - R_\tau^{-1} y_+)\frac{J}{y_+} - \left(\frac{J}{y_+}\right)^2 \tag{8}$$

The solutions of the functional equations. (5)–(7) mainly depend on the choice of functions J and K. That Izakson [1] and Millikan [2] for fully developed turbulent pipe or channel flow adopted $J = 1/\kappa$ (where κ is the Karman universal constant), leading to log laws for velocity in traditional overlap region. In the present work we consider flow in the buffer layer, a cushion between sublayer and fully developed turbulent flow domain [1,2] and outer layer flow.

2.1. Inner most log law for velocity profile; Karman log law of buffer layer

The functional Eq. (6) with $J = A_i = 1/\kappa_i$ yields velocity profile as another log law

$$u_+ = A_i \ln y_+ + B_i \tag{9}$$

which was first proposed by Karman [13] and Afzal [14,15] in the buffer layer with $A_i = 1/\kappa_i = 5.03$ and $B_i = -3.05$. In this case Reynolds stress from Reynolds momentum equation (1) becomes

$$\tau_+ = 1 - \frac{A_i}{y_+} - Y \tag{10}$$

The turbulent energy production (8) from (9) and (10) becomes

$$P_+ = (1 - R_\tau^{-1} y_+)\frac{A_i}{y_+} - \left(\frac{A_i}{y_+}\right)^2 \tag{11}$$