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The 35-Moment System with the Maximum-entropy Closure for Rarefied Gas Flows

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Abstract

This paper presents a robust implementation of the maximum-entropy closure in the context of rarefied gas dynamics. In contrast to the closure of Grad, moment systems supplied with the maximum-entropy closure are hyperbolic in the interior of the domain of definition, allowing applications to strongly non-equilibrium gas flows.

The 35-moment system includes as basis functions all monomials up to order four, so that evolution equations for important non-equilibrium quantities, such as the stress tensor and heat flux vector are included in the system. We consider a fixed, block-wise Gauss-Legendre quadrature rule for the numerically approximate moments of the reconstructed maximum-entropy distribution function. The convex dual optimization problem is solved with a Newton type algorithm. We show that the Hessian matrix used in the Newton iteration can become ill-conditioned even for equilibrium states if monomial basis functions are used. Therefore, we consider partially and fully adaptive basis algorithms to improve the robustness of the optimization algorithm and demonstrate that the 35-moment system allows for accurate and robust simulations of non-equilibrium flows by applying the model to one-dimensional gas processes, such as a continuous shock-structure problem and a Riemann initial value problem.

Keywords:
maximum-entropy, moment equations, shock wave structure, two-beam problem
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1. Introduction

We consider moderately rarefied gas flows in the transition regime, for which the Knudsen number Kn, defined as the ratio between the mean free path, λ_0, of a particle and a characteristic macroscopic length scale, l_0, is of order unity. Compared to the continuum regime (Kn $\ll 1$), strong non-equilibrium effects can occur in the transition regime, which invalidate the assumptions of the constitutive laws of Navier-Stokes-Fourier, such that more sophisticated models are necessary for an accurate description of moderately rarefied gas flows, see e.g. [1].

Moment equations provide a flexible framework for the derivation of evolution equations for macroscopic quantities from the Boltzmann equation. One of the simplest moment models for gases leads to the Euler system, which describes a gas in the local equilibrium state. The inclusion of evolution equations for higher-order moments, such as the stress tensor and heat flux vector, can extend the validity of this model from the continuum regime into the transition regime, see e.g. [2, 3, 4].

Moment systems are not in closed form and require an additional constitutive law. In the context of moment equations these laws are known as moment closure theories. Entropy-based moment closures [5, 6, 7] lead to symmetric hyperbolic partial differential equations. In contrast to Grad's classical closure [2, 8], the hyperbolicity of the equations is not lost for states far from equilibrium, so that entropy-based closures can be used for the simulation of strongly non-equilibrium gas processes, such as shock-structure calculations. First numerical results to the 14-moment system with the maximum-entropy closure and boundary conditions have been reported in [9] for a shock wave problem and a Couette flow, which are in very good agreement with the reference Boltzmann solution obtained by DSMC.

In [1] an explicit, closed-form closure for the 14-