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Natural vibration of an aqueous pendant drop

Lauren M. Parkinson, Chi M. Phan*

Department of Chemical Engineering, Curtin University, Perth, WA 6845, Australia

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A B S T R A C T

Interfacial dynamics of a pendant drop suspended from a needle support was studied via a new experimental method, without external excitation, to accurately quantify the impact of size and surface properties on the natural frequency. For drops ranging between 3.0 and 40.2 mm^3, the natural frequency of vibration varied proportionally to the volume raised to the exponent −3/4, $V^{-3/4}$. A direct relationship between the frequency and the water drops was obtained $f(Hz) \propto S^{-3/4} t_c^{-1}$, in which S is the dimensionless size parameter. Previous works performed employing external excitation suggested a direct impact of the supporting size, which might be explained by the acting force. The current experimental results do not follow this prediction, indicating insignificant influence of the support base on the recorded frequency. The results indicate a significant role of surface tension, which is taken into account by size factor S, on the vibration of the pendant drop.

1. Introduction

Drops are commonly observed to oscillate and deform in natural and industrial processes. Whilst large drops typically fragment due to aerodynamic forces, smaller drops often remain intact, suggesting a dependence on their size and surface tension. The natural vibrations of the water droplets have been investigated for over a century. However, the understanding of the underlying phenomena remains unsatisfactory. This study addresses the deficit in knowledge by applying a novel method with the use of a high-speed camera and mathematical interpretation.

1.1. Research significance

Growing interest in the field of drop dynamics and vibrational response characteristics may be attributed to its practical applications in predicting the physical mechanisms involved in a range of scientific and engineering based systems. Drop vibration influences a number of processes including spray cooling and coating, inkjet printing [1], electric field enhanced liquid-liquid extraction, humidification vibration induced drop atomisation (VIDA) [2–4] or ejection [5,6] and internal mixing and de-mixing of multi-component systems [7–10].

The dynamic behaviour of drops and bubbles plays a pivotal role in a number of key processing equipment, particularly those involving multi-phase systems such as chemical reactors and separation equipment [2]. Heat and mass transfer mechanisms between the drops, the bubbles and the continuous phase, play a governing role in the efficiency of such equipment, and thus by increasing the fundamental understanding of their respective dynamics may unveil a mechanism for increasing the aforementioned transport rates [2].

Additionally, liquid drops excited at their resonant frequency are understood to easily disengage from the attached substrate, thus pulsed ejection of liquid drops via externally applied electrical excitation may aid in increasing the efficiency of spray or dispersion equipment provided accurate determination of the dominant resonant frequency [11]. Liquid ejection may be theoretically replicated via the study of pendant drops suspended from a needle (thin nozzle), to assess the dynamic properties and modal frequencies [12]. Such equipment, however, is typically influenced by the effects of volume and liquid properties including viscosity and surface tension. Therefore, developing a greater understanding of the vibrational tendencies in relation to various drop parameters holds potential for increasing the efficiency and capabilities of a suite of chemical processing technologies.

1.2. Theoretical background

The oscillation of a drop is a fundamental phenomenon in fluid studies. Early works of Plateau [13], Rayleigh [14] and Kelvin [15], have explored the vibrational frequency of capillary waves for freely oscillating inviscid spherical drops under the action of interfacial tension. Via a linear energy analysis method, Rayleigh derived an expression for the fundamental angular frequency (ω) as a function of the drop radius (R), fluid density (ρ), surface tension (γ) and mode number (n = 2, 3, 4), as per the following generalised equation [10]:

* Corresponding author.
E-mail address: c.phan@curtin.edu.au (C.M. Phan).

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$$f_n^2 = \frac{n(n-1)(n+2)\gamma}{4\pi^2\rho R^3} \quad (1)$$

Thus, for a purely spherical water drop, the frequency (f) of mechanical oscillations was found to be directly proportional to the radius raised to the power of negative three on two ($-3/2$) [16]:

$$f_n \propto R^{-\frac{3}{2}} \quad (2)$$

Lamb [17] extended from Rayleigh's linear analysis to investigate the oscillations of viscous spheres embedded within an external inviscid fluid, assessing the effects of surface tension and density effects at the drop interface. Lamb generalised Rayleigh's theoretical equation for angular frequency to include the influence of a host medium possessing non-negligible density in the absence of gravity.

For the characterisation of liquid-liquid systems, Lamb's frequency is commonly used in the literature for theoretical and experimental comparisons. For example, in 1949, Gunn indicated [18] that microfluid drops behaved within close agreement to Rayleigh's theory, predicting that a logarithmic plot of frequency versus drop size will return a directly proportional decrease [18], later confirmed by the works of Brook and Latham [19].

Strictly speaking however, the Rayleigh—Lamb theory predicts that the oscillatory frequency of the $n = 1$ mode (displacement), has zero frequency (degenerated) for a free drop. Thus, in the case of an externally applied force, such as gravity, the drop will proceed to accelerate without returning back. Hence, whilst the Rayleigh—Lamb theory is applicable for free drops, the formula cannot be applied for constrained drops in partial or total contact with a solid support.

Frequency of small amplitude oscillations for drop systems of finite viscosity may be determined by the non-linear dispersion relation proposed by Miller and Scriven [20], expressed by the vanishing of the determinant of a 7×7 matrix. Notably, however, the system of equations cannot be solved in general case. Instead, analytical solutions were obtained for a selection of key limiting cases [2,20].

More recently, the oscillation of a spherical droplet in contact with a solid base has been investigated theoretically [21,22]. The results indicated that the constraint (being the area of contact with a solid base [23]) can regulate the vibration frequency. It is noteworthy that these theoretical analyses focused on a spherical shape.

1.3. Experimental studies on vibration

For isolated droplet, experiments have been carried out in the immiscible liquid-liquid systems, in which the falling velocity of the drop is considerably reduced, thus enabling frequency analysis. For instance, experimental studies by Loshak and Byers [24] and Ramabhadran et al. [25] of various organics falling through an aqueous organic phase returned an accuracy of within 10 per cent when compared with the theoretical predictions [26].

More recently, there has been growing interest in the vibrational analysis of attached drops, such as sessile drops supported on a flat surface [27], or pendant drops hanging from a nozzle support [2]. The attached drops have more practical application [28].

Analytical treatments [29,30] of drops partially constrained on a spherical bowl shaped solid support are probably the most comprehensive works regarding the vibratory nature of attached drops. Strani and Sabetta demonstrated that for a supported drop there exists an additional oscillatory mode ($n = 1$) corresponding to the net movement of the drops centre of mass, a situation that is logically infeasible for an isolated drop [31]. Notably, whilst Strani and Sabetta had assumed a spherical support shape in the interest of convenience, although being more restrictive when compared to a flat support [30], experimental work carried out by Rodot et al. [32] demonstrated negligible influence of the support shape on the vibrational frequency.

For a hanging pendant drop, however, the droplet deforms significantly from the spherical shape due to the increased ratio between gravity and surface tension. In this case, the vibration is resultant effect of deformation and elasticity of the necking region near solid support. DePaoli et al. [2] studied forced oscillations of a pendant drop suspended from a nozzle, reporting on frequency-drop size correlations. The authors concluded that the non-spherical shape of the drop profile profoundly influenced the oscillatory response, which deviates from the theoretical prediction for a spherical shape [30]. The results from different parameters indicated that the dynamics of such pendant–shape oscillations are governed by four dimensionless groups:

- Reynolds number (*Re*),
- Gravitational Bond number (*Bo*),
- A size factor (for example, the ratio between equivalent radius and nozzle radius), and
- A shape factor (for example, the ratio between vertical and horizontal dimensions of the droplet).

The Reynolds number was defined via a characteristic time, which was calculated from droplet size, density and surface tension. The Bond number is defined from gravity, surface tension and droplet size. In this analysis, the shape and size factors are thus inter-dependent for the same drop volume. It should be noted that comprehensive experiment covering wide range of the above four factors are have not been obtained due to experimental difficulties.

Importantly, majority of the experimental work on hanging pendant droplets has been carried out to assess the natural vibrations via a method of forced vibrations, typically through external acoustic [33], electrical or mechanical excitations [10]. In these experiments, the frequency which caused the greatest deformation of the drop profile was consequently recorded as the natural frequency. It has been argued that the recorded frequency was not significantly influenced by the means by which the oscillations were induced [34]. However, the external excitations are typically applied through the support base, and thus are directly proportional to the contact area. Hence, the external excitation may induce artificial effects on the resultant frequency. For instance, it has been demonstrated that the excitation amplitude can have a substantial influence on the resonance frequency at high *Re* [28].

In summary, the most effective experimental works rely on a type of attached drop. However, to easily observe the natural frequency, the current experimental methodology relies on a means of external excitation to enhance the oscillatory response. Application of such an external force has the ability to interfere with the natural oscillations, particularly as any applied force is proportional to the contact area between the drop and the solid support. The current work aims to experimentally obtain the natural frequency without the application of any external perturbations. Consequently, the results can confirm, or otherwise, the influence of the support base on the natural vibration.

2. Methodology

Building on the previous pendant drop method [35], the current experimental work aims to bridge the gap in fundamental understanding, to devise a generalised equation that summarises the dominant natural frequency of pendant drops without externally applied vibrations. The current experimental work aims to explore the application of high-speed/high-resolution photography, by studying the small scale, finite oscillations of pendant drops without the addition of any external perturbations.

Consequently, via the use of high-speed/high-resolution digital photography, the current work aims to assess the replicability of previous analytical and experimental conclusions by analysing the oscillatory behaviour of pendant drops as a standing wave.