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Tidal bore progressing on a small slope

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ABSTRACT

In a natural estuary, a tidal bore may progress on a small sloping bed from the downstream to the upstream. In this study, a simple analytical solution for tidal bore formed in a small slope channel was developed using the finite control volume analysis. New unsteady experiments were conducted to verify the theoretical model. The model predictions generally agree with the observations. A general relation is obtained for the conjugate depth ratio as a function of the Froude number and the channel slope from the experimental data. The results indicate that the conjugate depth ratio increases with an increasing Froude number as well as with a decrease in channel slope. On a negative slope, the Froude number increases as the bore propagates along the channel, and decreases for a positive slope. The theoretically based model is accurate and simple to estimate the celerity of the tidal bore progressing along a small slope channel.

1. Introduction

A tidal bore is a special geophysical phenomenon in which the leading edge of the flood tide forms an undular or breaking bore that travels up a river or narrow bay against the direction of the initial flow current [1]. An undular bore is a positive surge characterised by a train of secondary waves following the surge front [2]. A breaking bore is a wall of turbulent water rushing upstream along the river with its foaming front and rumble noise [3]. A tidal bore is a moving hydraulic jump. This problem was studied by scientific researchers and hydraulic engineers for a couple of centuries. Using the shallow-water equations, Barré de Saint-Venant [4] first predicted the theoretical development of a tidal bore. Other theoretical analysis and literature reviews comprise [5–7]. Although most studies considered horizontal channels, the bed of natural estuaries with tidal bore generally presents some slope. In the Qiantang River, China, the bore results from the funnel-shaped character of the Hangzhou Bay and a sand bar that occupies the mouth [8–10]. The rising sand bar with an average riverbed slope of 0.0002 decreases the water depth, and the funnel-shaped bay concentrates the water energy as well, resulting in the strong Qiantang River tidal bore.

While hydraulic jumps on sloping channels have been studied [11–13], there is limited research about the tidal bore progressing on a slope [14]. Combining a theoretical derivation and new physical data, the tidal bore progressing on a small slope with different slope angle θ ($-0.004 < \theta < 0.004$) is investigated here.

2. Theoretical models for a small slope channel

Let us consider a tidal bore progressing upstream on a prismatic channel with small slope θ depicted in Fig. 1. Take a fixed and deforming control volume with length L and width B, between an upstream section 1 and the end of the channel section 2. In Fig. 1, d, V and P are the flow depth, velocity and water pressure, respectively; f is the boundary shear force; G is the gravity force; C, t and L are respectively the celerity, progressing time and distance of the tidal bore; d_j is the conjugate water depth, that is the flow depth immediately behind the bore front; the subscripts 1 and 2 refer to the flow conditions at sections 1 and 2 (Fig. 1a).

For a small slope θ ($-0.004 < \theta < 0.004$), the control volume $\bar{V}$ is (Fig. 1a)

$$\bar{V} = B(\Omega_1 + \Omega_2) \tag{1}$$

where $\Omega_1 = d_1(L - Ct) - 0.5m(L - Ct)^2$, $\Omega_2 = d_2Ct + 0.5mC^2t^2$, $m = \tan\theta + \tan\varphi$. φ is the friction slope [2]

$$\tan\varphi = \frac{4\tau_0}{\rho g D} \tag{2}$$

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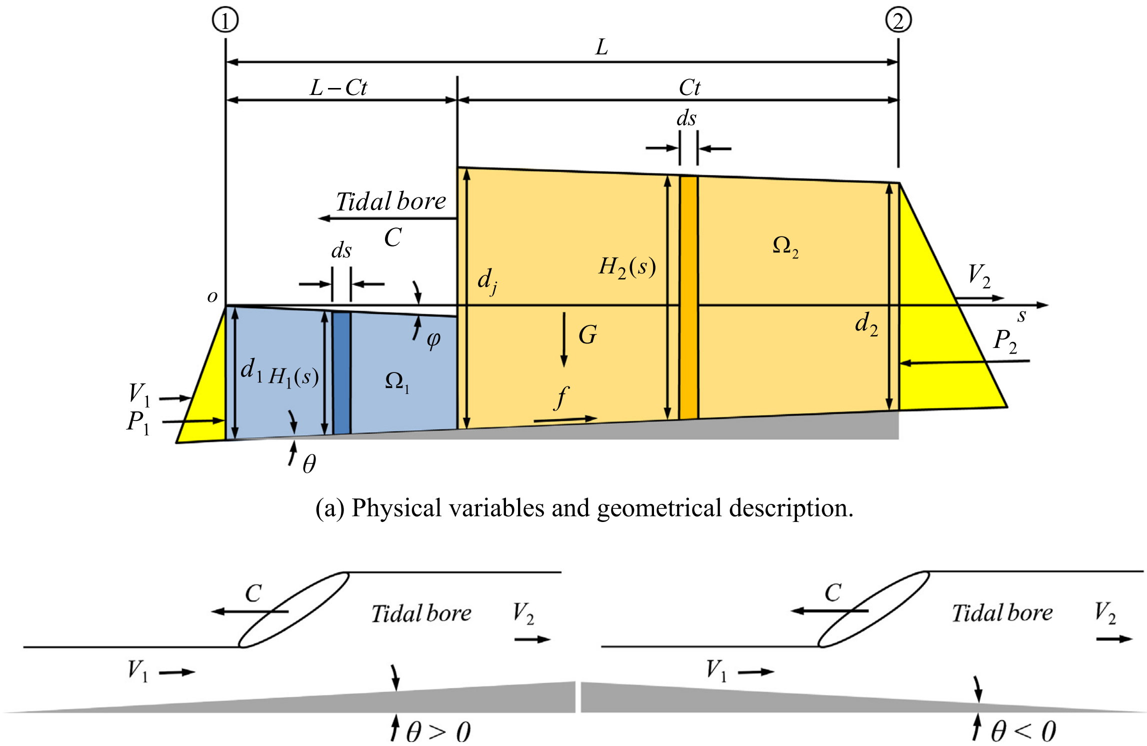

(a) Physical variables and geometrical description.

(b) Downward propagation. (c) Upward propagation.

Fig. 1. Sketch of tidal bore progressing upstream along a prismatic channel. (a) Physical variables and geometrical description. (b) Downward propagation. (c) Upward propagation.

where τ_0 is the boundary shear stress, $\tau_0 = \lambda\rho V^2/8$, λ is the Darcy-Weisbach friction factor, and D is the hydraulic diameter.

2.1. Conservation of mass

For the control volume (CV), the equation of conservation of mass may be expressed as

$$\frac{d}{dt}\int_{CV}\rho d\bar{V} = -\int_{Ain}\rho(\vec{V}\cdot\vec{n})dA - \int_{Aout}\rho(\vec{V}\cdot\vec{n})dA \tag{3}$$

where ρ is the density of water; $\vec{V}$ is the velocity vector; $\vec{n}$ is the normal vector; and $\rho(\vec{V}\cdot\vec{n})dA$ is the mass flow rate through the differential area dA. The sign of the dot product $\vec{V}\cdot\vec{n}$ is "+" for flow out of the control volume and "−" for flow into the control volume.

Assuming ρ to be a constant and considering the velocity distribution at the sections 1 and 2 are uniform, then $\int_{Ain}\rho(\vec{V}\cdot\vec{n})dA = -B\rho V_1 d_1$, and $\int_{Aout}\rho(\vec{V}\cdot\vec{n})dA = -B\rho V_2 d_2$.

From Eq. (1), it yields

$$\frac{d}{dt}\int_{CV}\rho d\bar{V} = \rho BC(d_2 - d_1 + Lm) \tag{4}$$

Substituting Eq. (4) to Eq. (3), it gives

$$d_2(V_2 + C) = d_1(V_1 + C) - CLm \tag{5}$$

2.2. The linear momentum equation

From Newton's second law, the linear momentum equation may be written in an integral form as

$$\frac{d}{dt}\int_{CV}\rho\vec{V}d\bar{V} = \sum\vec{F}_{ex} - \int_{Ain}\rho(\vec{V}\cdot\vec{n})\vec{V}dA - \int_{Aout}\rho(\vec{V}\cdot\vec{n})\vec{V}dA \tag{6}$$

where $\sum\vec{F}_{ex}$ is the total external forces acting on the control volume; $\int_{Ain}\rho(\vec{V}\cdot\vec{n})\vec{V}dA = -\rho BV_1V_1d_1$, and $\int_{Aout}\rho(\vec{V}\cdot\vec{n})\vec{V}dA = -\rho BV_2V_2d_2$.

From Eq. (1), it yields

$$\frac{d}{dt}\int_{CV}\rho\vec{V}d\bar{V} = \rho B\frac{d}{dt}\int_{\Omega_1}\vec{V}dA + \rho B\frac{d}{dt}\int_{\Omega_2}\vec{V}dA \tag{7}$$

The conservation of mass implies that the discharge any section in Ω_1 and Ω_2 is equal, i.e., $q_1(s) = V_1d_1$, $q_2(s) = V_2d_2$, where $q_1(s)$ and $q_2(s)$ are the discharge per meter width at section 1 and 2, respectively. Based upon geometric considerations (Fig. 1):

$$\int_{\Omega_1}\vec{V}dA = \int_0^{L-Ct}V(s)H_1(s)ds = \int_0^{L-Ct}q_1(s)ds = V_1d_1(L-Ct) \tag{8}$$

$$\int_{\Omega_2}\vec{V}dA = \int_{L-Ct}^{L}V(s)H_2(s)ds = \int_{L-Ct}^{L}q_2(s)ds = V_2d_2(L-Ct) \tag{9}$$

Substituting Eqs. (8) and (9) into Eq. (7), it yields

$$\frac{d}{dt}\int_{CV}\rho\vec{V}d\bar{V} = \rho BC(V_2d_2 - V_1d_1) \tag{10}$$

The total external force acting on the control volume shown in Fig. 1 along the surface of the slope is

$$\sum\vec{F}_{ex} = P_1 + P_2 + f + G\sin\theta \tag{11}$$