International Communications in Heat and Mass Transfer 87 (2017) 91–97

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

A novel non-iterative inverse method for estimating boundary condition of the furnace inner wall

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ARTICLE INFO

Keywords:
Non-iterative inverse method
Least-square error method
Inverse boundary conditions
Boundary element method
Furnace inner wall

ABSTRACT

A novel non-iterative inverse method with two-step scheme based on boundary element method (BEM) is proposed to estimate the boundary condition of furnace inner wall. Firstly, a matrix equation of heat transfer is formed by BEM with some measurement points. Then, the boundary value of discrete nodes on the furnace inner wall can be solved by the least-square error method. Different shapes of the furnace inner wall are considered with different function expressions, where some factors are discussed to validate the performance of present method such as the measured error, the number of measurement points, the distributive position of measurement points and the mixed boundary conditions. The results of numerical examples show that the present method can obtain the great performance on both the inversing accuracy and the computing efficiency even for the large heat flux.

1. Introduction

In general, the inverse heat conduction problems mainly include the identification of the material properties [1–3], the geometry shapes [4] and the boundary conditions [5,6] where the boundary conditions are generally required accurate knowledge in many industrial applications. Conventionally, these boundary conditions were obtained by expensive experimental methods with delicate equipment. However, in recent years, the studies of inverse numerical algorithms have achieved great development. It is well known that the conjugate gradient method [7–9] is one of the most widely used in inverse problems. Moreover, there are Levenberg–Marquardt method [10,11], the Tikhonov regularization method [12,13], the genetic algorithm [14,15] and the linear least-squares error method [16], and so on. It is noted that the solving processes of direct problems will be called many times until the condition of convergence is satisfied in these inverse procedure.

For the direct problems of heat transfer, there are many typical numerical methods to solve through several decades of development such as finite element method (FEM), finite difference method (FDM), meshless method and boundary element method (BEM). Especially, BEM is one of the most popular numerical methods for the heat transfer problems [17–19].

In inverse problems fields based on BEM, Huang and his co-workers have obtained a series of achievements. For example, Huang and Chao [20] derived firstly the 2D steady formulation for determining the unknown irregular boundary configurations with CGM. Huang and Tsai [21] extended the method to solve 2D transient shape identification problems. Afterward, Huang et al. [22] developed a modified model which can estimate 2D multiple cavities with the steepest descent method (SDM). Also Huang and Chen [23] estimated the transient boundary conditions by CGM. In recent years, many researchers also solved many inverse problems based on BEM which involve the identification of crack [24], the identification of parameters of piezoelectric material [25], the inverse boundary condition in elasticity [26], identification of 3D thermal conductivity coefficients of anisotropic media [27], and the estimation of unknown boundary shape of fluid-solid conjugate heat transfer problem [28], and so on. Compared with FEM, meshless methods and other numerical methods in solving inverse heat transfer problem, BEM has a particular advantage for solving the large gradient problems. However, the iteration process in these literatures is an essential part. Different initial values of iteration and searching step sizes can directly influence the accuracy and efficiency of results. Though BEM only needs to discretize the boundary of problem, the degree of freedom is seemingly smaller than the method of domain discretizing such as FEM. But it is generally known that the matrix of coefficient of BEM is a full matrix. Therefore, the computational costs are also increased because the direct process will be called many times for solving the inverse problems by the iteration process.

There is a non-iterative inverse method which can quickly solve the inverse problems besides the iteration method. Certainly, the non-

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http://dx.doi.org/10.1016/j.icheatmasstransfer.2017.06.017

Nomenclature

N	number of node
T	temperature
$\mathbf{T}$	temperature vector
$\mathbf{Q}$	heat flux vector
x_1, x_2	Cartesian coordinate of node $\mathbf{x}$

Greek symbols

Ω	domain of problem
Γ	boundary of the domain Ω
ε	standard deviation
$\boldsymbol{\omega}$	random vector

Superscripts

T	transpose of vector

Subscripts

b	boundary node
e	estimation value
i	inner boundary
o	outer boundary

Abbreviations

RMS	root mean square

iterative inverse method also must depend on some information from the direct problem. The direct process is solved just only several times generally no more than three times. In 2007, Su and Chen [29] presented a method to identify the geometry of furnace inner wall on the basis of an inverse matrix and a virtual area concept, where the direct problem was solved by FDM. Then the method was applied to estimate the inner boundary of furnace with two layer walls [30]. In 2016, Yu et al. [31] adopted BEM and the concept of virtual boundary to identify the geometry boundary of the furnace inner wall. But the inverse matrix of transformation will become high ill-posed when the scheme is used to solve the relative complex problems.

To the authors' knowledge, this work is the first application of inverse the boundary conditions based on BEM. In this paper, a new non-iterative scheme is proposed to inverse the boundary condition of any shape with different kinds of functions for two-dimension steady heat conduction problems. In the presented method, the node temperatures on the inner boundary are obtained by the rearrangement of coefficient matrix.

The remainder of the text is structured as follows. First, the description of direct problems is given in Section 2.1. Section 2.2 then introduces the detailed inverse scheme. After that, the several numerical examples are shown in Section 3. The paper ends with some concluding remarks.

2. Description of direct problems and inverse scheme

2.1. The direct problem

The inverse schemes are established by the formulation of direct problems in this paper. As shown in Fig. 1, a two-dimension furnace wall is considered, where the outer boundary Γ_o of the furnace wall is a circle of radius r_o and the inner boundary Γ_i can be any shape. Here, the temperature in furnace is regarded as steady after a long time of combustion and the thermal conductivity of furnace wall is assumed to be constant. For expressing convenience, an inner boundary of random shape with the temperature boundary condition is used to describe the inverse procedure. As shown in Fig. 1, the outer boundary Γ_o is imposed the temperature $\overline{T_0}$ and the inner boundary Γ_i is imposed the temperature $\overline{T_i}$. Certainly, the boundary conditions are not only restricted the first kind temperature condition. The other kinds of boundary conditions can be also used in present paper.

The governing equation is given as

$$\nabla^2 T(\mathbf{x}) = 0 \quad , \quad \mathbf{x} \in \Omega \tag{1}$$

where $\mathbf{x} = (x_1, x_2)$ is a point in the actual solved domain Ω and ∇^2 is Laplace operator. The boundary conditions can be expressed as

$$T(\mathbf{x}) = \overline{T}_o \quad , \quad \mathbf{x} \in \Gamma_o \tag{2}$$

$$T(\mathbf{x}) = \overline{T}_i \quad , \quad \mathbf{x} \in \Gamma_i \tag{3}$$

It is well known that Eqs. (1)–(3) the boundary value problem can be quickly solved by BEM [32]. After discretizing the boundary integral equation by using the linear element, the linear equations can be expressed as

$$\begin{cases} \mathbf{H}_b\mathbf{T}_b = \mathbf{G}_b\mathbf{Q}_b & \mathbf{x} \in \Gamma_o \cup \Gamma_i \\ \mathbf{T}_i = \mathbf{G}_i\mathbf{Q}_b - \mathbf{H}_i\mathbf{T}_b & \mathbf{x} \in \Omega \end{cases} \tag{4}$$

where the vector $\mathbf{T}_i$ is the temperatures of internal nodes, the vectors $\mathbf{T}_b$ and $\mathbf{Q}_b$ are the temperatures and heat fluxes of boundary nodes, respectively. The coefficient matrices $\mathbf{H}_b$, $\mathbf{G}_b$, $\mathbf{G}_i$ and $\mathbf{H}_i$ are generated from the boundary integral equations. It is noted that the subscript "b" and "i" denote which the point $\mathbf{x}$ locates on the boundary and interior of domain, respectively.

According to the first formula in Eq. (4), the heat fluxes of boundary nodes can be written as

$$\mathbf{Q}_b = \mathbf{G}_b^{-1}\mathbf{H}_b\mathbf{T}_b \tag{5}$$

Substituting Eq. (5) into the second formula in Eq. (4), the temperature of internal nodes can be expressed as

$$\mathbf{T}_i = (\mathbf{G}_i\mathbf{G}_b^{-1}\mathbf{H}_b - \mathbf{H}_i)\mathbf{T}_b \tag{6}$$

2.2. The inverse theory and procedure

In general, the iteration is an essential procedure for inversing the boundary condition problems. However, the present method does not needs iteration procedure as mentioned in the introduction. The unknown boundary conditions will be directly obtained in the following expression.

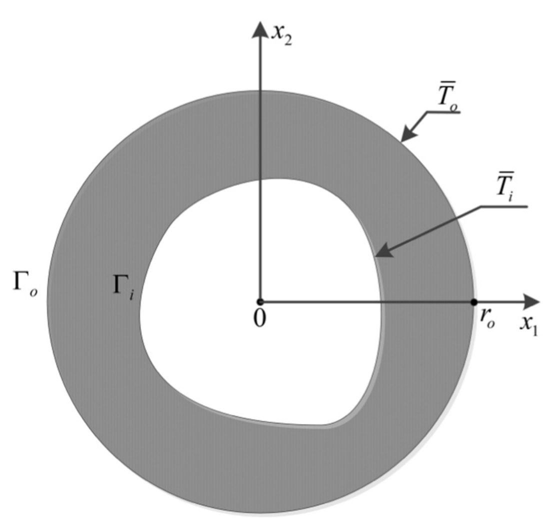

Fig. 1. The furnace wall model.