



# Inverse estimation of the unknown base heat flux in irregular fins made of functionally graded materials



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## ABSTRACT

In this study, an inverse algorithm based on the conjugate gradient method and the discrepancy principle has been successfully applied to an irregular fin made of functionally graded materials to estimate the unknown base heat flux distributions by using temperatures at some measurement locations. The inverse results, in which three different base heat flux distributions are to be determined, have proven current method's capability to accurately estimate arbitrary fin-base heat flux distributions even measurement errors have been taken into account. The temperature data calculated from the direct problem are used to simulate the measured temperature. The influence of measurement errors upon the precision of the estimated results is also investigated. This method does not need any prior information on the unknown quantity, and results show that excellent estimations can be obtained for the test cases considered in this study.

## 1. Introduction

Given their capability to largely increase heat transfer area, fins have been widely used to enhance heat transfer rate in many engineering problems. They work to conduct heat from a heat source to their surfaces where heat can be dissipated through convection or radiation to the surrounding fluid or environment. At present, they often serve as vital heat transfer components in power generators, semiconductors, exothermic reactors, and many others devices where excessive heat has to be removed to keep these devices function normally. Due to their importance in engineering, fins have been the subjects of many studies, for example Refs. [1–4], and a good portion of these studies focused on understanding the heat transfer characteristics of one or two-dimensional fins in steady or transient heat transfer problems.

To optimize thermal resistance and temperature profile in solid structures under heavy thermal loading, composite materials made by composition of several different components are often employed to meet demanding requirements. Such ingenious idea of making composite materials was first proposed by Japanese researcher [5] and has evolved into the concept of functionally graded materials (FGMs). In FGMs, transition from one material to another is in a gradual manner so that local stress concentration caused by abrupt change in material properties can be eliminated. Hence, FGMs are far more promising than traditional composite materials to meet strict demands under extreme

thermal conditions in some industries. For example in aerospace industry, they have been proposed to be used in future high-speed air vehicles where temperature resistant and light weight structure will be required. FGMs' superior performance to resist high temperature is rooted in their excellent thermo-mechanical properties, which make them also suitable for applications in nuclear reactors, pressure vessels and pipes, and chemical plants [6–8]. In these applications, a thin shield made of FGMs is able to cope with elevated temperature gradients without developing excessive thermal stresses, thus increasing their reliability. Another advantage of FGMs can be found in heat exchanger pipes and high-temperature engine parts where mismatch of thermal expansion coefficient can be avoided by a continuous transition from ceramic to metallic materials.

After a few decades' development, inverse analysis has gained significant progress and is now widely used to solve many engineering problems. When applied to heat transfer problems, inverse analysis can be used to estimate temperature, heat flux, heat transfer coefficient, or even materials' thermophysical properties such as thermal conductivity and heat capacity [9,10]. Furthermore, it is also commonly used to design special geometries to meet specific thermal requirements or problems involving shape identification [11,12]. Most of the studies on inverse heat conduction problems in open literature dealt with homogeneous materials. Only very few have focused on problems involving FGMs. For example, Golbahar Haghighi et al. [13] applied conjugate gradient method (CGM) on a two-dimensional transient inverse heat

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Nomenclature		$y_0$	half height of the fin (m)
$h$	convective heat transfer coefficient ( $\text{W m}^{-2} \text{K}^{-1}$ )	<i>Greek symbols</i>	
$J$	functional	$\Delta$	small variation quality
$J'$	gradient of functional	$\beta$	step size
$k$	thermal conductivity ( $\text{W m}^{-1} \text{K}^{-1}$ )	$\gamma$	conjugate coefficient
$k_0$	thermal conductivity at the fin tip ( $\text{W m}^{-1} \text{K}^{-1}$ )	$\varepsilon$	convergence criterion
$L_1$	distance from central line to the tip (m)	$\lambda$	variable used in the adjoint problem
$L_2$	distance from central line to the base (m)	$\sigma$	standard deviation
$M$	number of measurement points	$\varpi$	random variable
$P$	direction of descent	$\Gamma_1, \Gamma_2$	fin boundaries
$Q$	base heat flux of the fin ( $\text{W m}^{-2}$ )	<i>Superscripts/subscripts</i>	
$T$	fin temperature ( $^\circ\text{C}$ )	$K$	iterative number
$T_\infty$	ambient temperature ( $^\circ\text{C}$ )		
$x, y$	spatial coordinate (m)		
$x_m$	$x$ coordinate of the temperature sensors (m)		
$Y$	measured temperature ( $^\circ\text{C}$ )		

conduction problem (IHCP) where the solid material is made of FGMs. One of the focuses of this study is to understand the effects of measurement errors on the accuracy of inverse solutions. Lee et al. [14] reported an inverse algorithm to estimate transient fin base heat flux using temperatures taken inside the fin which is also made of FGMs. They obtained good agreement between the inverse and exact heat fluxes. Wang et al. [15] conducted an inverse study in estimating the unknown inner wall geometry of a FGM furnace. This is the so called “shape identification problem”. They also investigated the effects of measurement errors and measurement locations on the accuracy of inverse solutions.

In the present study, a CGM inverse algorithm for estimating the unknown heat flux at the base of an irregular fin from the knowledge of temperature measurements taken within the fin has been developed and tested. The irregular fin is made of FGMs, and the base heat flux is assumed steady. Hence, this is a steady-state IHCP problem for a FGMs medium and on an irregular geometry. CGM is originated from the perturbation principles and transforms the inverse problem into the solutions of three problems, direct, sensitivity, and adjoint [16–18]. The detail algorithm of current CGM will be discussed in the following sections.

## 2. Analysis

### 2.1. Direct problem

The following 2-D fin problem is considered here to illustrate the methodology for developing expressions for the use in estimating the unknown base heat flux of an irregular fin, based on the temperature measurements at some measurement positions. Fig. 1 shows the configuration for an irregular fin considered in this study. Since the irregular fin geometry is symmetrical to  $x$ -axis, only one half of the fin is needed for computation domain. The mathematical formulation of this 2-D irregular fin problem is given as follows [19,20]:

$$\frac{\partial}{\partial x} \left[ k(x) \frac{\partial T(x,y)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ k(x) \frac{\partial T(x,y)}{\partial y} \right] = 0, \tag{1}$$

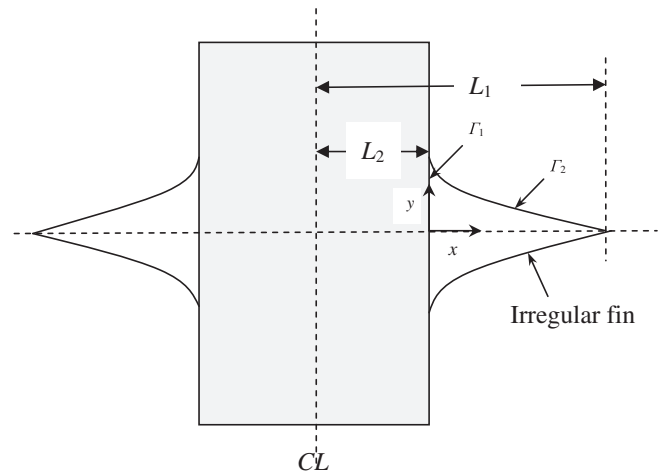
$$-k(x) \frac{\partial T(x,y)}{\partial n} = Q(y), \text{ at } \Gamma_1, \tag{2}$$

$$-k(x) \frac{\partial T(x,y)}{\partial n} = h(s)[T(x,y) - T_\infty], \text{ at } \Gamma_2, \tag{3}$$

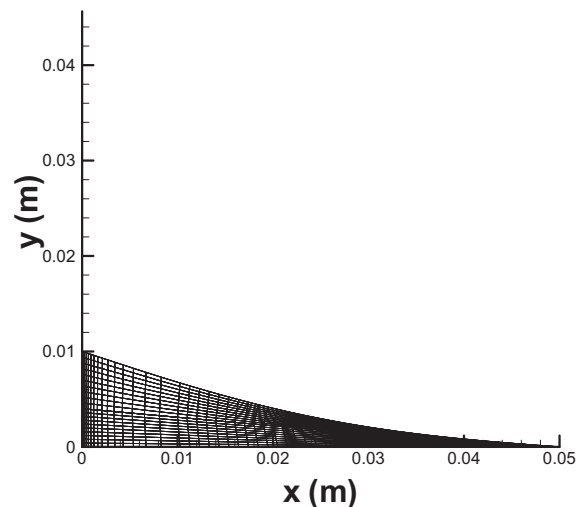
$$\frac{\partial T(x,y)}{\partial n} = 0, \text{ at } y = 0. \tag{4}$$

Here,  $k(x)$  is the thermal conductivity,  $Q(y)$  is the unknown base heat

flux,  $h(s)$  is the convective heat transfer coefficient, and  $T_\infty$  represents the ambient temperature of the fin. Unlike the standard heat conduction analysis, which assumes the material of fin to be homogeneous with uniform material properties, the present analysis assumes that the



(a) Configuration of the current problem.



(b) Mesh with 1000 cells.

Fig. 1. Configuration and computational mesh of the present irregular fin problem.

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