International Journal of Heat and Fluid Flow 63 (2017) 190–204

Contents lists available at ScienceDirect

International Journal of Heat and Fluid Flow

journal homepage: www.elsevier.com/locate/ijhff

An elliptic blending differential flux model for natural, mixed and forced convection

F. Dehoux [a,b], S. Benhamadouche [a], R. Manceau [b,c,*]

[a] EDF R&D, MFEE Dept., 06 quai Watier, Chatou 78400, France
[b] Institute Pprime, Dept. Fluid flow, heat transfer and combustion, CNRS-Univ. of Poitiers-ENSMA, SP2MI, téléport 2, 11 bd Marie et Pierre Curie, BP 30179, 86962 Futuroscope Chasseneuil Cedex, France
[c] Dept. of mathematics and applied mathematics, Inria-Cagire group, CNRS-university of Pau, IPRA, avenue de l'université, BP115, 64013 Pau Cedex, France

A R T I C L E I N F O

Article history:
Available online 7 October 2016

Keywords:
Turbulence modelling
Turbulent heat flux
Buoyancy
Wall blockage
Natural convection
Mixed time scale

A B S T R A C T

Several modifications are introduced to the Elliptic Blending Differential Flux Model proposed by Shin et al. (2008) to account for the influence of wall blockage on the turbulent heat flux. These modifications are introduced in order to reproduce, in association with the most recent version of the EB-RSM, the full range of regimes, from forced to natural convection, without any case-specific modification. The interest of the new model is demonstrated using analytical arguments, *a priori* tests and computations in channel flows in the different convection regimes, as well as in a differentially heated cavity.

1. Introduction

Many industrial applications involving heat or mass transfer phenomena, in particular in the field of energy production, are still treated with linear eddy-viscosity models and the Simple Gradient Diffusion Hypothesis (SGDH) to model the turbulent heat fluxes. Within this class of models, one of the most successful approaches in forced convection flows (e.g., Parneix et al., 1998; Manceau et al., 2000; Sveningsson and Davidson, 2005; Billard and Laurence, 2012) is the elliptic relaxation concept, under the form of its eddy-viscosity version, the V2F model, originally developed by Durbin (1991), or one of its stabilized formulations (Hanjalić et al., 2004; Laurence et al., 2005), combined to a SGDH approach with a constant turbulent Prandtl number. For buoyant flows, a step further in the sophistication of the elliptic relaxation models was made by Kenjereš et al. (2005), who introduced an algebraic flux model (AFM) with a buoyancy-extended V2F model. However, it is generally admitted that the Reynolds-stress models are desirable (Hanjalić and Launder, 2011) for mixed and natural convection, due to the presence of significant anisotropic phenomena.

In the last few years, the Elliptic Blending Reynolds-Stress Model (EB-RSM, Manceau and Hanjalić, 2002), has emerged as a numerically robust alternative to the elliptic relaxation concept, in particular for isothermal and forced convection applications (e.g., Thielen et al., 2005; Borello et al., 2005; Viti et al., 2007; Billard et al., 2011). The Generalized Gradient Diffusion Hypothesis (GGDH, Daly and Harlow, 1970) proved sufficient to model the turbulent heat fluxes in the absence of buoyancy, due to the correct reproduction of turbulence anisotropy in the near-wall region by the EB-RSM. For the mixed and natural convection regimes, transposing the model of Manceau and Hanjalić (2002) for the Reynolds stresses into a model for the turbulent heat fluxes, Shin et al. (2008) proposed a differential flux model (DFM) based on the elliptic blending strategy to account for the near-wall region. However, as will be shown in Section 4, this model is not fully satisfactory in the natural convection regime, which led (Choi and Kim, 2008) to modify the coefficients of the EB-RSM in order to improve the predictions, at the expenses of the predictions in forced convection. The present work aims at developing a modified EB-DFM that can be used in association with the most recent version of the EB-RSM (Manceau, 2015) in the full range of regimes, from forced to natural convection, without any case-specific modification.

2. The elliptic blending strategy

The Reynolds-stress transport equation reads

$$\frac{\partial \rho\, \overline{u_i' u_j'}}{\partial t} + \frac{\partial \rho \overline{u}_k\, \overline{u_i' u_j'}}{\partial x_k} = \rho\left(P_{ij} + D_{ij}^{\nu} + D_{ij}^{t} + \phi_{ij}^{*} - \varepsilon_{ij} + G_{ij}\right) \qquad (1)$$

where P_{ij}, D_{ij}^{ν}, D_{ij}^{T}, ϕ_{ij}^{*} and ε_{ij} stand for the production, the molecular diffusion, the turbulent diffusion, the velocity-pressure gradient correlation and the dissipation tensors, respectively.

* Corresponding author. Fax: +33559407555.
E-mail address: remi.manceau@univ-pau.fr (R. Manceau).

http://dx.doi.org/10.1016/j.ijheatfluidflow.2016.09.003

$G_{ij} = -g_i\beta\overline{u'_j\theta'} - g_j\beta\overline{u'_i\theta'}$ is the production term arising from buoyancy forces, assuming a linear variation of density with temperature. Note that, throughout the present paper, the Boussinesq approximation is used, i.e., the density variations are only accounted for in buoyant terms and the velocity field is divergence-free.

In order to account for the effects of wall blockage on turbulence (Manceau, 2015), in the EB-RSM, the difference $\phi^*_{ij} - \varepsilon_{ij}$ is formulated as a blending

$$\phi^*_{ij} - \varepsilon_{ij} = (1-\alpha^3)(\phi^w_{ij} - \varepsilon^w_{ij}) + \alpha^3(\phi^h_{ij} - \varepsilon^h_{ij}), \tag{2}$$

of a quasi-homogeneous model $\phi^h_{ij} - \varepsilon^h_{ij}$ (i.e., a model not valid in the near-wall region and requiring the use of wall functions), herein the SSG model (Speziale et al., 1991); and the near-wall model given by

$$\phi^w_{ij} = -5\frac{\varepsilon}{k}\left[\overline{u'_iu'_k}n_jn_k + \overline{u'_ju'_k}n_in_k - \frac{1}{2}\overline{u'_ku'_l}n_kn_l\left(n_in_j + \delta_{ij}\right)\right], \tag{3}$$

$$\varepsilon^w_{ij} = \frac{\overline{u'_iu'_j}}{k}\varepsilon. \tag{4}$$

The blending function α^3 is related to the distance-to-the-wall-sensitive function α, solution of the elliptic relaxation equation:

$$\alpha - L^2\nabla^2\alpha = 1, \tag{5}$$

which is zero at the wall (Dirichlet boundary condition) and goes to unity far from the wall. The unit vector $\mathbf{n}$ is a generalization of the notion of wall-normal vector: $\mathbf{n} = \nabla\alpha/\|\nabla\alpha\|$.

The dissipation equation reads

$$\frac{D\varepsilon}{Dt} = \frac{C'_{\varepsilon_1}P - C_{\varepsilon_2}\varepsilon}{\tau} + \frac{\partial}{\partial x_l}\left(\frac{C_\mu}{\sigma_\varepsilon}\overline{u'_lu'_m}\,\tau\,\frac{\partial\varepsilon}{\partial x_m}\right) + \nu\frac{\partial^2\varepsilon}{\partial x_k\partial x_k}, \tag{6}$$

where τ is Durbin's time scale

$$\tau = \max\left(\frac{k}{\varepsilon}, C_T\sqrt{\frac{\nu}{\varepsilon}}\right). \tag{7}$$

The variable $C'_{\varepsilon1}$ coefficient

$$C'_{\varepsilon_1} = C_{\varepsilon_1}\left[1 + A_1\left(1-\alpha^3\right)\frac{P}{\varepsilon}\right] \tag{8}$$

is intended to represent the term $P_{\varepsilon3}$ in the exact ε-equation (Hanjalić and Launder, 2011), which stimulates the production of dissipation in the buffer layer (Mansour et al., 1988). Since the initial proposal of Manceau and Hanjalić (2002), the model has undergone numerous modifications. The full set of equations and coefficients is given in Appendix A. For a justification of the version used in the present work, the reader is referred to Manceau (2015).

For mixed and natural convection, elaborate heat flux models are needed in order to account for buoyancy/turbulence interactions (Hanjalić, 2002). Shin et al. (2008) and Choi and Kim (2008) proposed extensions of the elliptic blending strategy to full differential flux models (DFMs), in order to account for the influence of the wall on the turbulent heat flux. With the objective of avoiding the resolution of additional transport equations, Dehoux et al. (2012) derived an *implicit* algebraic version of such models, called the elliptic-blending algebraic flux model (EB-AFM), which is merely a near-wall extension of the standard AFM (Dol et al., 1997). Recently, Vanpouille et al. (2014) derived an *explicit* algebraic heat flux model using the elliptic blending strategy, and successfully computed buoyant flows in mixed and natural convection regimes.

A DFM consists in closing the Reynolds-averaged temperature equation by solving the transport equation for the turbulent heat flux,

$$\frac{D\rho\,\overline{u'_i\theta'}}{Dt} = \rho\left(P_{i\theta} + G_{i\theta} + \phi^*_{i\theta} - \varepsilon_{i\theta} + D^\nu_{i\theta} + D^t_{i\theta}\right). \tag{9}$$

In this equation, the production terms $P_{i\theta}$ and $G_{i\theta}$ do not require modelling, contrary to the scrambling term $\phi^*_{i\theta}$, the dissipation term $\varepsilon_{i\theta}$ and the turbulent and molecular diffusion terms $D^t_{i\theta}$ and $D^\nu_{i\theta}$. Shin et al. (2008) and Choi and Kim (2008) applied the elliptic blending strategy (see Eq. 2) to the scrambling and dissipation vectors,

$$\phi^*_{i\theta} = \left(1-\alpha^n_\theta\right)\phi^w_{i\theta} + \alpha^n_\theta\phi^h_{i\theta},$$
$$\varepsilon_{i\theta} = \left(1-\alpha^n_\theta\right)\varepsilon^w_{i\theta} + \alpha^n_\theta\varepsilon^h_{i\theta}, \tag{10}$$

where $\alpha_\theta = \alpha$ is chosen as the same blending function as used in Eq. (2), and $n = 2$. Similar to the case of the Reynolds stresses, this approach makes possible the extension to the near-wall region of quasi-homogeneous models $\phi^h_{i\theta}$ and $\varepsilon^h_{i\theta}$. Assuming the isotropy of the small scales, $\varepsilon^h_{i\theta} = 0$ is imposed; for the scrambling term $\phi^h_{i\theta}$, the standard quasi-homogeneous model

$$\phi^h_{i\theta} = -C_{1\theta}\frac{1}{\tau}\overline{u'_i\theta'} + C_{2\theta}\overline{u'_j\theta'}\frac{\partial\overline{u_i}}{\partial x_j} + C_{3\theta}\beta g_i\overline{\theta'^2} \tag{11}$$

is used. Shin et al. (2008) and Choi and Kim (2008) used the coefficients proposed by Launder (1988) and Peeters and Henkes (1992), respectively.

In order to satisfy the asymptotic near-wall behaviour of the difference $\phi^w_{i\theta} - \varepsilon^w_{i\theta}$, Shin et al. (2008) showed that the two terms can be written as

$$\phi^w_{i\theta} = -\left[1 + \frac{\gamma_1}{2}\left(1 + \frac{1}{Pr}\right)\right]\frac{1}{\tau}\overline{u'_k\theta'}n_kn_i \tag{12}$$

and

$$\varepsilon_{i\theta} = \frac{1}{2}\left(1 + \frac{1}{Pr}\right)\frac{1}{\tau}\left(\overline{u'_i\theta'} + (1-\gamma_1)\overline{u'_k\theta'}n_kn_i\right). \tag{13}$$

Note that the asymptotic analysis leading to these equations is based on the common assumption that the fluctuations of the wall temperature are negligible due to the thermal inertia of the solid material. The additional complexity due to conjugate heat transfer when the conduction in the solid cannot be neglected (Flageul et al., 2015; Tiselj et al., 2001), or simply due to an imposed heat flux at the wall, is not addressed herein and is left to future work. Here, the difference $\phi^w_{i\theta} - \varepsilon^w_{i\theta}$, and consequently, the results, are independent of the particular value of γ_1, since only this difference is involved in Eq. (9). Shin et al. (2008) used $\gamma_1 = 1$, whereas, in order to satisfy the individual asymptotic behaviour of $\phi^w_{i\theta}$ and $\varepsilon^w_{i\theta}$, Choi and Kim (2008) used $\gamma_1 = 0$.

Turbulent and molecular diffusion terms are modelled as

$$D^t_{i\theta} = \frac{\partial}{\partial x}\left(C_\theta\overline{u'_ku'_l}\tau\frac{\partial\overline{u'_i\theta'}}{\partial x_l}\right) \tag{14}$$

(Daly and Harlow, 1970) and

$$D^\nu_{i\theta} = \frac{\partial}{\partial x_k}\left(\frac{\kappa+\nu}{2}\frac{\partial\overline{u'_i\theta'}}{\partial x_k} + \gamma_2 n_in_j\frac{\nu-\kappa}{6}\frac{\partial\overline{u'_j\theta'}}{\partial x_k}\right), \tag{15}$$

respectively. Choi and Kim (2008) and Shin et al. (2008) used $\gamma_2 = 1$ (Shikazono and Kasagi, 1996) and $\gamma_2 = 0$ (Peeters and Henkes, 1992), respectively.

The transport equation for the temperature variance is simply modelled as

$$\frac{\partial\rho\overline{\theta'^2}}{\partial t} + \frac{\partial\left(\rho\overline{u_j}\;\overline{\theta'^2}\right)}{\partial x_j} = \frac{\partial}{\partial x_k}\left[\left(\rho\kappa\delta_{kl} + C_\theta\rho\,\tau\,\overline{u'_ku'_l}\right)\frac{\partial\overline{\theta'^2}}{\partial x_l}\right] + \rho P_\theta - \rho\varepsilon_\theta, \tag{16}$$