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Technical Note

An improved lattice Boltzmann method for solid-liquid phase change in porous media under local thermal non-equilibrium conditions

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ABSTRACT

This paper presents an improved lattice Boltzmann (LB) method to simulate solid-liquid phase change with natural convection in porous media under local thermal non-equilibrium (LTNE) conditions. In this method, three distribution functions are respectively adopted for flow field, and temperature field of the PCM and solid matrix. Different from previous models, the present model for temperature field incorporates the total enthalpy and a free parameter in the equilibrium distribution function, and thus could have high computational efficiency by avoiding iteration procedure to deal with phase change. The present model is validated by the melting with natural convection in a square cavity filled with a metal foam. It is found that the numerical results are in good agreement with other numerical results, and the present method could preserve higher accuracy due to numerical diffusion reduction through keeping the relaxation time at around unity as well as tuning the free parameters properly.

1. Introduction

Solid-liquid phase change in porous media is frequently encountered in lots of natural and engineering systems [1]. Over the past several decades, this problem has been extensively investigated analytically, experimentally and numerically [1]. In particular, many numerical methods have been adopted to simulate this problem [1,2]. However, mathematical modeling of such problem is still a challenging task because of the nonlinear characteristics of phase change process, as well as the complexity of porous structure.

As a powerful numerical tool based on the kinetic theory, the lattice Boltzmann (LB) method has been applied to the fluid flow and heat transfer [3–8]. In recent years, the LB method has been already adopted to model solid-liquid phase change in porous media not only under local thermal equilibrium (LTE) conditions [9–11] but also under local thermal non-equilibrium (LTNE) conditions [12,13]. Based on LTNE condition, Gao et al. developed a thermal LB model with three distribution functions [12]. Tao et al. applied LB method to study the melting processes in the metal foam/paraffin composite PCMs [13]. However, these existing LB models under LTNE conditions need multiple iterations to deal with phase change, which may increase the computational cost, due to the fact that non-linear latent-heat source term in the energy equation is treated as the corresponding source in LB equation. In addition, the use of the models with single relaxation time (SRT) collision approach, could encounter numerical diffusion across phase interface when the relaxation time is not unity [14]. For reducing the numerical diffusion in simulating phase change problem without porous media, the multi-relaxation-time (MRT) approach is employed by Huang and Wu [14]. However, compared with the MRT model, the SRT model has become the most widely used form of the LB model because of its simplicity and computational efficiency. Therefore, it is important to develop an improved SRT model which can reduce numerical diffusion across phase interface.

In this work, we aim to develop an improved LB model for simulating the solid-liquid phase change problems in porous media under LTNE conditions, which can overcome the above-mentioned difficulties. In the present model, by introducing the total enthalpy into the equilibrium distribution function for the PCM temperature, the non-linear latent-heat source term in the LB equation vanished and thus phase change can be tackled without iteration procedure. In addition, through tuning the free parameter properly, the relaxation time can be fixed at around unity to reduce numerical diffusion across phase interface for modeling the PCM temperature field.

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2. Mathematical formulation

With the assumption of local thermal non-equilibrium between the solid matrix and the PCM, the volume average energy equations for solid-liquid phase change in porous media can be described as follows [1,2]:

$$\varepsilon\frac{\partial[(\rho c_p)_f T_f]}{\partial t}+\nabla\cdot[(\rho c_p)_{fl}\boldsymbol{u}T_f]=k_{e,f}\nabla^2T_f+h_V(T_s-T_f)-\varepsilon\rho_{fl}L_a\frac{\partial f_l}{\partial t}, \tag{1}$$

$$(1-\varepsilon)\frac{\partial[(\rho c_p)_s T_s]}{\partial t}=k_{e,s}\nabla^2T_s+h_V(T_f-T_s), \tag{2}$$

where $\boldsymbol{u}$, T, ε, ρ, c_p, L_a, f_l, k_e and h_V are the velocity, temperature, porosity, density, specific heat, latent heat of melt, liquid fraction of the PCM, equivalent thermal conductivity and volumetric heat transfer coefficient, respectively; the subscripts s, f and fl refer to the solid matrix, the PCM, and the liquid phase of the PCM. The volumetric heat transfer coefficient h_V is the product of interfacial heat transfer coefficient between PCM and porous media and specific surface area of porous matrix. Eqs. (1) and (2) can also written in the form:

$$\varepsilon\frac{\partial En_f}{\partial t}+\nabla\cdot[(\rho c_p)_{fl}\boldsymbol{u}T_f]=k_{e,f}\nabla^2T_f+h_V(T_s-T_f), \tag{3}$$

$$(1-\varepsilon)\frac{\partial En_s}{\partial t}=k_{e,s}\nabla^2T_s+h_V(T_f-T_s), \tag{4}$$

where the total enthalpy of the PCM, En_f, is given as $En_f=(\rho c_p)_f T_f+\rho L_a f_l$, and En_s is defined as $En_s=(\rho c_p)_s T_s$. In addition, the volume average velocity, $\boldsymbol{u}$, satisfies the continuity equation and the Brinkman-Forchheimer equation [1,2], which are given as

$$\nabla\cdot\boldsymbol{u}=0, \tag{5}$$

$$\frac{\partial\boldsymbol{u}}{\partial t}+(\boldsymbol{u}\cdot\nabla)\left(\frac{\boldsymbol{u}}{\varepsilon}\right)=-\frac{1}{\rho_{fl}}\nabla(\varepsilon p)+\nu_e\nabla^2\boldsymbol{u}+\boldsymbol{F}, \tag{6}$$

where p and ν_e are the volume average pressure, respectively. F represents the total body force due to the presence of porous media and other external force, and can be expressed as

$$\boldsymbol{F}=-\frac{\varepsilon\nu_{fl}}{K}\boldsymbol{u}-\frac{\varepsilon F_\varepsilon}{\sqrt{K}}|\boldsymbol{u}|\boldsymbol{u}+\varepsilon\boldsymbol{g}\beta(T_f-T_{ref}), \tag{7}$$

where ν_{fl} is the kinematic viscosity of the fluid, K is the permeability, F_ε is the Forchheimer form coefficient, β is thermal expansion coefficient, T_{ref} denotes the reference temperature, and $\boldsymbol{g}$ is the gravity acceleration. The fluid flow and heat transfer, can be characterized by some dimensionless parameters: the Prandtl number Pr, the viscosity ratio J, the Rayleigh number Ra, the Darcy number Da and the Stefan number Ste, and the interstitial Nusselt number, which are defined as follows:

$$Pr=\frac{\nu_f}{\alpha_f},J=\frac{\nu_e}{\nu_{fl}},Ra=\frac{g\beta\Delta TL^3}{\nu_{fl}\alpha_{fl}},Da=\frac{K}{L^2},Ste=\frac{c_{p,f}\Delta T}{L_a},\quad Nu_d=\frac{h_V d_p^2}{k_{fl}}, \tag{8}$$

where ΔT represents the characteristic temperature difference, L denotes the characteristic length and d_p is the pore diameter.

3. Lattice Boltzmann model

3.1. Lattice Boltzmann equation for the flow field in porous media

The LB equation for the velocity field is written as [4]:

$$f_i(\boldsymbol{r}+\boldsymbol{e}_i\delta t,t+\delta t)-f_i(\boldsymbol{r},t)=-\frac{1}{\tau}[f_i(\boldsymbol{r},t)-f_i^{eq}(\boldsymbol{r},t)]+\delta tF_i, \tag{9}$$

where f_i, is the distribution function for the velocity field with discrete velocity $\boldsymbol{e}_i$ at position $\boldsymbol{r}$ and time t; δt is the time step. f_i^{eq} in Eq. (9) is defined as [4]

$$f_i^{eq}=\omega_i\left(1-\frac{1}{2\tau}\right)\rho\left[1+\frac{\boldsymbol{e}_i\cdot\boldsymbol{u}}{c_s^2}+\frac{(\boldsymbol{e}_i\cdot\boldsymbol{u})^2}{2\varepsilon c_s^4}-\frac{\boldsymbol{u}^2}{2\varepsilon c_s^2}\right], \tag{10}$$

where ω_i and c_s are the weight coefficients and the sound speed. The discrete body force term F_i can be described as [4]

$$F_i=\omega_i\rho\left[\frac{\boldsymbol{e}_i\cdot\boldsymbol{F}}{c_s^2}+\frac{(\boldsymbol{u}\boldsymbol{F}:\boldsymbol{e}_i\boldsymbol{e}_i)}{\varepsilon c_s^4}-\frac{\boldsymbol{u}\cdot\boldsymbol{F}}{\varepsilon c_s^2}\right]. \tag{11}$$

The macroscopic fluid density can be calculated by $\rho=\sum_i f_i$. The fluid velocity $\boldsymbol{u}$ is computed using a temporal velocity $\boldsymbol{V}$ and is given as [4]

$$\boldsymbol{u}=\frac{\boldsymbol{V}}{d_0+\sqrt{d_0^2+d_1|\boldsymbol{V}|}},\quad \boldsymbol{V}=\sum_i\boldsymbol{e}_i f_i/\rho+\frac{\delta t}{2}\varepsilon[\boldsymbol{g}\beta(T_f-T_{ref})], \tag{12}$$

where the two parameters d_0 and d_1 are given $d_0=1/2(1+\varepsilon\delta t\nu_{fl}/(2K))$, $d_1=\varepsilon\delta tF_\varepsilon/(2\sqrt{K})$. In this work, we use the nine-velocity model in two dimensions (D2Q9 model). The dimensionless relaxation time is given as $\tau=\nu_e/(c_s^2\delta t)+0.5$.

3.2. Lattice Boltzmann equations for the temperature fields of the PCM and solid matrix

The SRT-LB equations for the temperature fields of the PCM and solid matrix are written as:

$$g_{i,f}(\boldsymbol{r}+\boldsymbol{e}_i\delta t,t+\delta t)-g_{i,f}(\boldsymbol{r},t)=-\frac{1}{\tau_{T,f}}[g_{i,f}(\boldsymbol{r},t)-g_{i,f}^{eq}(\boldsymbol{r},t)]+\left(\delta t+\frac{\delta t^2}{2}\partial_t\right)Sr_{i,f}+\delta tSu_{i,f}, \tag{13}$$

$$g_{i,s}(\boldsymbol{r}+\boldsymbol{e}_i\delta t,t+\delta t)-g_{i,s}(\boldsymbol{r},t)=-\frac{1}{\tau_{T,s}}[g_{i,s}(\boldsymbol{r},t)-g_{i,s}^{eq}(\boldsymbol{r},t)]+\left(\delta t+\frac{\delta t^2}{2}\partial_t\right)Sr_{i,s}, \tag{14}$$

where $\tau_{T,f}$ and $\tau_{T,s}$ are the dimensionless relaxation times of the PCM and solid matrix phases, respectively. Moreover, $g_{i,f}^{eq}$ and $g_{i,s}^{eq}$ are the equilibrium temperature distribution function of the PCM and that of solid matrix phase and can be expressed respectively as

$$g_{i,f}^{eq}=\begin{cases}\varepsilon En_f-\gamma_f T_f+\omega_i\gamma_f T_f, & i=0\\ \omega_i T_f(\gamma_f+(\rho c_p)_{fl}\frac{\boldsymbol{e}_i\cdot\boldsymbol{u}}{c_s^2}), & i\neq0\end{cases}, \tag{15}$$

$$g_{i,s}^{eq}=\begin{cases}(1-\varepsilon)En_s-\gamma_s T_s+\omega_i\gamma_s T_s, & i=0\\ \omega_i\gamma_s T_s, & i\neq0\end{cases}, \tag{16}$$

where γ_f and γ_s are the two free parameters, respectively, which keep unvaried over the entire space. The two discrete source terms, $Sr_{i,f}$ and $Sr_{i,s}$, can be defined respectively as $Sr_{i,f}=\omega_i h_V(T_s-T_f)$, and $Sr_{i,s}=\omega_i h_V(T_f-T_s)$. The discrete source term $Su_{i,f}$ is given as

$$Su_{i,f}=\omega_i\left(1-\frac{1}{2\tau_{T,f}}\right)\frac{\boldsymbol{e}_i}{c_s^2}\cdot\frac{\partial[(\rho c_p)_{fl}T_f\boldsymbol{u}]}{\partial t}. \tag{17}$$