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Passive scalar diffusion in the near field region of turbulent rectangular submerged free jets

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ABSTRACT

Jets are a common way to transfer mass among fluids, or from a fluid to a surface. At moderate Reynolds numbers and low turbulent intensities the jet exhibits a Near Field Region (NFR) several diameters long. The paper presents numerical results and a theoretical model for the passive scalar diffusion of a submerged free jet in the NFR. Large Eddy Simulations (LES), in the Reynolds number range of 5000–40,000 and the Schmidt number range 1–100, are performed obtaining the passive scalar fields. Three mathematical models for the passive scalar diffusion are presented; the first one is valid in the NFR, specifically in the Undisturbed Region of Flow (URF), and the other two, obtained under the hypotheses of Tollmien and Görtler momentum spreading, are valid in the Potential Core Region (PCR). The last two models employ a turbulent Schmidt number inversely proportional to the mean velocity gradient, conclusion obtained from the LES numerical results. The self-similar solutions of the passive scalar show good agreement with the LES results. The wide range of Reynolds and Schmidt numbers investigated gives generality to the results.

1. Introduction

Heat and mass transfer among free jets, and between a jet and a solid, or a liquid surface is controlled by the fluid dynamics of the jet. The Reynolds number (Re) and the turbulence intensity (Ti) have a great influence upon the dispersion of particles, or droplets, in jets [1–3], and the heat transfer from a jet to a solid surface [4–5]. In several applications of circular jets it is desirable to have high Reynolds number (Re > 40,000) and turbulence intensities (Ti > 5%) and the dispersed phase can be described by some simplified solutions, as in the Fully Developed Region (FDR), [6,7]. In other applications, such as micro-jets for drug injection, [8,9], and recent welding technologies, [10,11], low turbulence intensities (Ti < 5%), and moderate Reynolds numbers (Re < 20,000) are requested. The dispersed phase field appears to be complex, and there is no mathematical model describing it. One aspect, rarely investigated, is the role that molecular diffusion plays in the micro-jets injection. The diffusivity, Γ, of a particle is proportional to its radius, according to the Stoke-Einstein relation, [12], and the Cunningham empirical equation, [13], allows to conclude that typical values of the particle radius, $r \approx 10^{-7} - 10^{-5} m$, lead to a Schmidt number of air in the range from 1 to 100.

The evolution of a turbulent rectangular submerged free jet has been widely investigated in the literature in the last decades with the following conclusion. The jet interacts with the stagnant fluid just downstream the slot exit and two regions of flow are defined, [14], the closer to the slot exit is the Near Field Region (NFR) and the farther one is the Fully Developed Region (FDR). In the NFR, the region of mixing fluid surrounds the potential core region (PCR), where the velocity on the centerline maintains the same value of the slot exit. When the mixing region has penetrated into the centerline, the NFR ends and the FDR begins. Tollmien [15], and Görtler [16], studied theoretically the flow of turbulent rectangular submerged free jets proposing a self-similar evolution of the axial velocity in the PCR and FDR. The equations describing the velocity evolution were confirmed experimentally in [17–21] for the PCR, and in [22–25] for the FDR.

In the last few years the evolution with two regions of flow, suggested in [14], has been revised. In the average flow the NFR can be split into two separate regions: the Undisturbed Region of Flow (URF) and the PCR. The URF, reported experimentally between the slot exit and the PCR, [26–28], for moderate Reynolds numbers (Re < 20,000) and low turbulence intensity (Ti = 0–5%), has not been noticed neither commented. Several experiments on

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Nomenclature

D	diameter
f	instantaneous self-similarity function
F	mean self-similarity function
h	half-height of the slot
k	turbulent kinetic energy
P	mean static pressure
p	instantaneous static pressure
r	particle radius
S_{ij}	rate of shear tensor
t	time
U	mean axial velocity
u	instantaneous axial velocity
V	mean cross-stream velocity
v	instantaneous cross-stream velocity
x	axial coordinate
y	cross-stream coordinate

Dimensionless parameters

a	similarity coefficient
c	Tollmien coefficient
e	Görtler coefficient
$Pe = Re \times Sc$	Peclet number
$Re = \frac{U_{in} D_h}{\nu}$	Reynolds number
$Sc = \frac{\nu}{\Gamma}$	Schmidt number
Sc_T	Turbulent Schmidt Number
Ti	Turbulent intensity

Greek

α_G	passive scalar coefficient (Görtler-like)
α_T	passive scalar coefficient (Tollmien-like)
$\dot{\gamma}$	shear rate
Γ	Passive scalar molecular diffusivity
Δ	filter width
ζ	Tollmien self-similarity variable
η	self-similarity variable
K_Φ	passive scalar coefficient
μ	dynamic viscosity
ν	kinematic viscosity
ν_T	turbulent viscosity
ξ	Görtler self-similarity variable
τ_{ij}^R	Reynolds stress tensor
ϕ	instantaneous passive scalar
ϕ'^2	mean passive scalar variance
Φ	mean passive scalar
ψ	instantaneous stream-function
Ψ	mean stream-function

Subscripts

h	hydraulic
in	inlet
sgs	sub-grid scale
T	turbulent

rectangular free jets of air, conducted by Gori and coworkers, [29–34], in the average flow evolution, enabled to identify the main characteristics of the URF, i.e., that velocity and turbulence remain almost equal to those measured on the slot exit. Numerical investigations have been able to reproduce the URF since Gori et al. [35], undertake preliminary numerical investigations with the Reynolds Averaged Navier-Stokes (RANS) equations model. The URF has been investigated also with the Large Eddy Simulations (LES) approach, at several Reynolds numbers, in order to verify the occurrence of URF, [36,37]. One of the main achievement of the last two studies was to show that the velocity profile in the URF obeys to a self-similar law, different from those proposed by Tollmien [15], and Görtler [16], for the PCR.

The presence of the URF influences the diffusion of the passive scalar in turbulent jets, which has been widely studied, but only limited to the FDR. Despite recognizing the self-similarity of the passive scalar, a mathematical formulation of its diffusion is still missing. The only exceptions are the studies on the FDR of circular jets, [6,7], which constitute the base for the "Gaussian Plume Model" (GPM). Such a model, widely used in environmental applications, [38,39], was derived under the hypotheses of constant eddy diffusivities for both the momentum and the passive scalar. The lack of a self-similar solution for the passive scalar spreading is probably due to the difficulties in the modeling of the turbulent Schmidt number.

The present work studies the diffusion of a passive scalar in the NFR of turbulent submerged air jets, issuing from a rectangular nozzle. The Schmidt numbers investigated range from 1 to 100, while the Reynolds numbers from 5000 to 40,000. Since the Peclet number spans between 5000 and 4,000,000 a full three-dimensional (3D) simulation is unpractical, [40,41]. The present numerical investigation is limited to the NFR and the domain is considered two-dimensional, 2D, which is a common assumption for Direct Numerical Simulations (DNS), [42–46], and LES approach, [47,48], of rectangular jets. Indeed, the near-field transitional mixing in turbulent jets is dominated by 2D large-scale coherent structures, as pointed out in [49]. This approximation has been demonstrated valid also in [37], where the 3D results for the NFR are in good agreement with the 2D results of [36]. In analogy with [36], a top-hat velocity profile is imposed on the slot exit and air is assumed incompressible.

2. Numerical method

2.1. Governing equations

In the LES approach, the governing equations are derived by filtering the Navier-Stokes equations and the filtered variables have the superscript ~. Applying the filtering operation, the mass and momentum equations read:

$$\frac{\partial \tilde{u}_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\tilde{u}_i\tilde{u}_j + \left(\tilde{p} + \frac{2}{3}C_I\Delta^2\tilde{\dot{\gamma}}^2\right)\delta_{ij} - 2\left(\frac{1}{\mathrm{Re}} + C_S\Delta^2\tilde{\dot{\gamma}}\right)\tilde{S}_{ij}\right) = 0 \tag{2}$$

where u_i is the velocity vector, p the static pressure, δ_{ij} the identity tensor, S_{ij} the rate of shear tensor

$$\tilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right) - \frac{1}{3}\frac{\partial \tilde{u}_k}{\partial x_k}\delta_{ij} \tag{3}$$

The filtered shear rate is

$$\tilde{\dot{\gamma}} = \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}} \tag{4}$$

with the filter width given by

$$\Delta = \sqrt{\Delta_m \Delta_m} \tag{5}$$