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Technical Note

Variable property solution for the natural convection in horizontal concentric annuli: Effect of the temperature difference ratio

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A R T I C L E I N F O

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A B S T R A C T

The variable property solution for the natural convection in air-filled horizontal concentric annuli is obtained by using the variable property-based lattice Boltzmann flux solver. The effect of temperature difference ratio is discussed under the condition that the *Ra* and *Pr* numbers are 5×10^4 and 0.71, respectively. In the large-gap annulus, only one pair of counter-rotating eddies is formed, and the equivalent conductivity increases with temperature difference ratio. In the moderate-gap annulus, the flow patterns depend on temperature difference ratio, and the equivalent conductivity sometimes has a notable decrease as the eddy number changes. These demonstrate that temperature difference ratio should be considered as an important parameter in the study about flow instability behaviors and heat transfer characteristics under non-Boussinesq conditions.

1. Introduction

In the past few decades, the natural convection in horizontal annuli had attracted considerable attentions due to its wide application in various energy systems. Many efforts [1–13] were devoted to exploring the influence of various factors and parameters, such as the *Ra* and *Pr* numbers, the radius ratio, the eccentricity, and external magnetic field, on flow instability behaviors and heat transfer characteristics by adopting the well-established Boussinesq approximation. However, the Boussinesq approximation was invalidated as a large temperature difference ratio was imposed [14]. Many investigations [15–21] were performed to illustrate the importance of non-Boussinesq effects; nevertheless, only partial variation in fluid properties was considered.

Recently, Cao and Zhang [22] studied the natural convection in horizontal annulus using the variable property-based lattice Boltzmann flux solver (VPLBFS) [23], and demonstrated the necessity of considering the total variation in fluid properties by comparison between the variable property solution (VPS) and several commonly-reported solutions with partial variation in fluid properties. In the present study, the VPLBFS is used to obtain the VPS for the natural convection in horizontal annuli. The primary purpose is to discuss the influence of temperature difference ratio under the condition that the *Ra* and *Pr* numbers are given constants. It can be anticipated that the present study will provide more accurate and reliable information for engineering applications.

2. Problem formulation

In the present study, the air-filled horizontal annulus has isothermal walls, with the inner radius and outer radius denoted by R_i and R_o, respectively. The outer wall temperature T_o is 300 K in all cases, while the inner wall temperature T_i varies from one case to another, ranging from 303 K to 900 K. Initially, the fluid is static, with the temperature equal to T_o. Similar to available studies [14–16,22,23], T_o is chosen as the reference temperature T_{Ref}. Consequently, the temperature difference ratio ϕ_0, defined by $(T_i - T_o)/T_{\mathrm{Ref}}$, varies from 0.01 to 2.0. The governing equations can be written as follows [22,23],

$$\begin{cases} (\partial_t p + \mathbf{u} \cdot \nabla p) + \rho c_s^2 \nabla \cdot \mathbf{u} = 0 \\ \partial_t(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p + \nabla[\mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)] + F \\ \partial_t T + \mathbf{u} \cdot \nabla T = \nabla \cdot \left(\frac{\kappa}{\rho C_p} \nabla T\right) - \kappa \nabla \frac{1}{\rho C_p} \cdot \nabla T \end{cases} \quad (1)$$

where c_s is the speed of sound. F is the buoyancy force, defined by $F = \vec{g}(\rho - \rho_{\mathrm{Ref}})$. $\vec{g}$ is the acceleration due to gravity. The subscript "Ref" identifies quantities at T_{Ref}. The functions provided by Zhong et al. [14] are employed to describe the temperature dependence of dynamic viscosity μ, specific heat C_p and thermal conductivity κ. As to fluid density ρ, a computer program called PROPATH 12.1 [24] is adopted. The property variations are shown in Fig. 1. By

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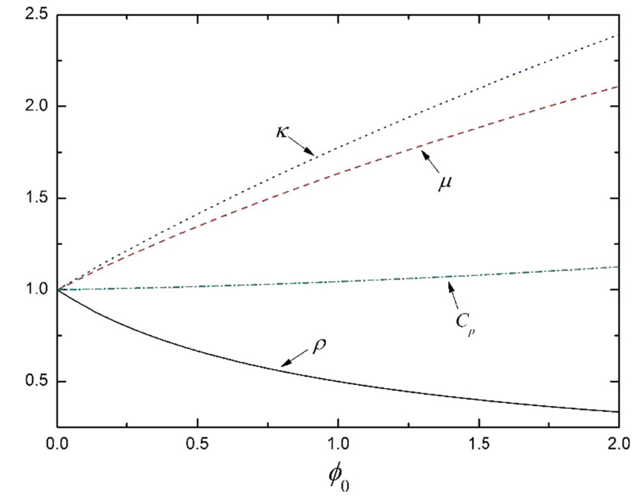

Fig. 1. The dimensionless function relationships between fluid properties and temperature. The fluid properties are normalized by their corresponding values at T_{Ref}.

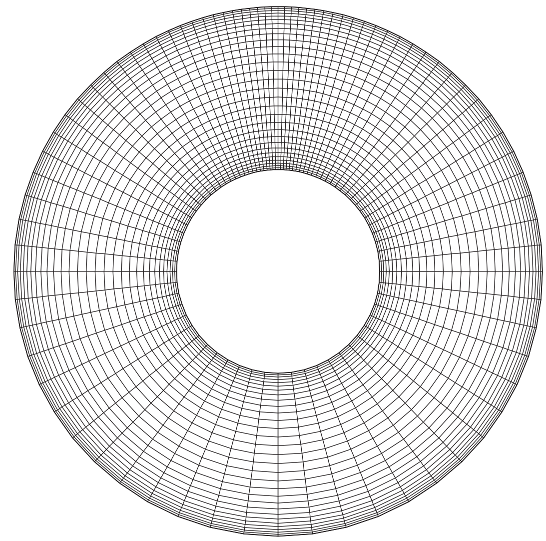

Fig. 2. The schematic diagram of non-uniform meshes.

nondimensionalizing Eq. (1), the definition formulas of the *Ra* and *Pr* numbers can be derived as follows [22],

$$Ra = \frac{g\Delta\rho L^3 \rho_{\mathrm{Ref}} C_{p\mathrm{Ref}}}{\mu_{\mathrm{Ref}}\kappa_{\mathrm{Ref}}}; \quad Pr = \frac{\mu_{\mathrm{Ref}} C_{p\mathrm{Ref}}}{\kappa_{\mathrm{Ref}}} \tag{2}$$

Here, $\Delta\rho$ is the magnitude of density difference corresponding to the temperature difference, $T_i - T_o$. L is the gap width of annulus, equal to $R_o - R_i$. In the present paper, the *Ra* and *Pr* numbers are set to be 5×10^4 and 0.71, respectively, and the radius ratio A, defined by R_o/R_i, takes two typical values of 2.6 and 1.4, respectively.

Table 1
Grid independence study for the natural convection with $\phi_0 = 0.5$.

A		Mesh numbers				
		40×20	80×30	120×30	120×40	160×50
2.6	$\overline{\kappa_{eqi}}$	4.120	4.093	4.096	4.098	4.097
	$\overline{\kappa_{eqo}}$	4.114	4.088	4.091	4.096	4.096
1.4	$\overline{\kappa_{eqi}}$	3.485	3.535	3.690	3.693	3.695
	$\overline{\kappa_{eqo}}$	3.485	3.535	3.688	3.692	3.694

3. Variable property-based lattice Boltzmann flux solver

The VPLBFS [22,23] is an efficient solver for low-speed thermal flow problems under non-Boussinesq conditions. The distribution function for pressure and momentum is denoted by $f(\mathbf{x},t)$, and that for temperature by $g(\mathbf{x},t)$. The most popular D2Q9 lattice velocity model is used, with the lattice velocity e_α taking the following form,

$$e_\alpha = \begin{cases} 0, & \alpha=0 \\ (\cos[(\alpha-1)\pi/2], \sin[(\alpha-1)\pi/2])c, & \alpha=1,2,3,4 \\ \sqrt{2}(\cos[(\alpha-5)\pi/2+\pi/4], \sin[(\alpha-5)\pi/2+\pi/4])c, & \alpha=5,6,7,8 \end{cases} \tag{3}$$

where $c = \delta_x/\delta_t$, with δ_t being the lattice time interval, equal to lattice constant δ_x. The sound speed c_s is given as $c_s = c/\sqrt{3}$. The equilibrium distribution functions, f_α^{eq} and g_α^{eq}, are given as follows,

$$f_\alpha^{eq}(\mathbf{x},t) = \omega_\alpha\left[p + \rho c_s^2\left(\frac{e_\alpha\cdot\mathbf{u}}{c_s^2} + \frac{(e_\alpha\cdot\mathbf{u})^2 - (c_s|\mathbf{u}|)^2}{2c_s^4}\right)\right], \quad \alpha = 0,1,2,\ldots,8 \tag{4a}$$

$$g_\alpha^{eq}(\mathbf{x},t) = \begin{cases} -\frac{3}{2}\omega_\alpha T\frac{|\mathbf{u}|^2}{c^2}, & \alpha=0 \\ \omega_\alpha T\left(\frac{3}{2} + \frac{3}{2}\frac{e_\alpha\cdot\mathbf{u}}{c^2} + \frac{9}{2}\frac{(e_\alpha\cdot\mathbf{u})^2}{c^4} - \frac{3}{2}\frac{|\mathbf{u}|^2}{c^2}\right), & \alpha=1,2,3,4 \\ \omega_\alpha T\left(3 + 6\frac{e_\alpha\cdot\mathbf{u}}{c^2} + \frac{9}{2}\frac{(e_\alpha\cdot\mathbf{u})^2}{c^4} - \frac{3}{2}\frac{|\mathbf{u}|^2}{c^2}\right), & \alpha=5,6,7,8 \end{cases} \tag{4b}$$

where ω_α is weighting coefficient, given as $\omega_0 = 4/9$, $\omega_1=\omega_2=\omega_3=\omega_4=1/9$ and $\omega_5=\omega_6=\omega_7=\omega_8=1/36$. The distribution functions and their equilibrium parts satisfy the following conservation relationships,

$$p = \sum_{\alpha=0}^{8} f_\alpha^{eq} = \sum_{\alpha=0}^{8} f_\alpha; \quad \rho\mathbf{u}c_s^2 = \sum_{\alpha=0}^{8} e_\alpha f_\alpha^{eq} = \sum_{\alpha=0}^{8} e_\alpha f_\alpha; \quad T = \sum_{\alpha=0}^{8} g_\alpha^{eq} = \sum_{\alpha=0}^{8} g_\alpha; \quad T\mathbf{u} = \sum_{\alpha=0}^{8} e_\alpha g_\alpha^{eq} = \sum_{\alpha=0}^{8} e_\alpha g_\alpha \tag{5}$$

By means of multi-scale Chapman-Enskog expansion analysis, the equations to be solved by the VPLBFS [22,23] can be obtained as follows,

$$\begin{cases} \frac{\partial p}{\partial t} + \nabla\cdot\left(\sum_{\alpha=0}^{8} e_\alpha f_\alpha^{eq}\right) = -\mathbf{u}\cdot\nabla(p - \rho c_s^2) \\ \frac{\partial\rho\mathbf{u}c_s^2}{\partial t} + \nabla\cdot\left[\sum_{\alpha=0}^{8} e_\alpha e_\alpha\left[f_\alpha^{eq} + (1 - \frac{1}{2\tau_f})f_\alpha^{neq}\right]\right] \\ = \nabla\cdot\left[\frac{\mu}{\rho}(\mathbf{u}\nabla + (\mathbf{u}\nabla)^T)(p - \rho c_s^2)\right] + Fc_s^2 \\ \frac{\partial T}{\partial t} + \nabla\cdot\left[\sum_{\alpha=0}^{8}(e_\alpha g_\alpha^{eq} + \left(1 - \frac{1}{2\tau_g}\right)e_\alpha g_\alpha^{neq})\right] = -\kappa\nabla T\nabla\frac{D}{\kappa} \end{cases} \tag{6}$$

where τ_f and τ_g are relaxation times, which can be calculated locally by Eq. (7).

$$\tau_f = \frac{\mu}{\rho c_s^2\delta_t} + 0.5; \quad \tau_g = \frac{3\kappa}{2\rho C_p c^2\delta_t} + 0.5 \tag{7}$$

The non-equilibrium parts, f_α^{neq} and g_α^{neq}, can be evaluated respectively from f_α^{eq} and g_α^{eq} at cell interface $(\mathbf{x},t)$ and its surrounding points $(\mathbf{x} - e_\alpha\delta_t, t - \delta_t)$ as follows,

$$f_\alpha^{neq}(\mathbf{x},t) = -\tau_f[f_\alpha^{eq}(\mathbf{x},t) - f_\alpha^{eq}(\mathbf{x} - e_\alpha\delta_t, t - \delta_t)] \tag{8a}$$

$$g_\alpha^{neq}(\mathbf{x},t) = -\tau_g[g_\alpha^{eq}(\mathbf{x},t) - g_\alpha^{eq}(\mathbf{x} - e_\alpha\delta_t, t - \delta_t)] \tag{8b}$$