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Optimization of the one-dimensional transient heat conduction problems using extended entransy analyses

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ABSTRACT

Heat transfer optimization principle is critically important for further explaining the underlying mechanisms and guiding practical designs of heat transfer processes. Recently, the entransy theory has been successfully used to optimize various steady-state heat transfer processes. Nevertheless, it is still an open question whether this theory can be utilized in transient cases. Here, we examined the applicability of the entransy analyses on the one-dimensional transient heat conduction process. It was found that the entransy dissipation rate can neither derive the transient governing equation nor correspond to the optimal result in the transient optimization problem. Therefore, an extended entransy dissipation rate was defined as the convolution integral of heat flux and negative temperature gradient. The total extended entransy dissipation rate over the time and space domain can correspond to the optimal result of the transient optimization problem. Additionally, Fourier transform was used to convert the transient problem from the time domain into the frequency domain, and the total entransy dissipation rate in the frequency domain will give a convenient optimization criterion that the temperature gradient field should be spatially uniform to reach the shortest characteristic time. Also, the inverse Fourier transform of the entransy dissipation rate in the frequency will be the extended entransy dissipation rate in the time domain. Finally, these findings were used to optimize a practical transient heat conduction problem for a solid thermal energy storage unit.

1. Introduction

Heat transfer is the exchange of thermal energy between physical systems due to a temperature difference with many engineering applications, such as in refrigeration [1] and energy storage systems [2]. Heat transfer processes should be optimized for efficient, sustainable energy utilization. The heat transfer governing equations are well established with many methods developed to improve energy generation, consumption, and conservation based on energy analyses [3]. However, heat transfer optimization still needs more in-depth investigations to further explain the underlying mechanisms and guide practical designs.

Bejan [4] proposed the entropy generation minimization principle for heat transfer optimization, i.e., the best heat transfer rate corresponds to the minimum entropy generation rate. Although the entropy generation rate has been widely used in heat transfer optimization analyses [5–9], several counter examples to the entropy generation minimization principle have been identified [10–13]. This is because the total entropy generation rate cannot be used to construct the least action principle of a heat transfer process [14]. Minimization of the total entropy generation rate cannot recover the fundamental heat transfer constitutive relation, Fourier's law [15–17]. The constitutive relation between the temperature gradient and the heat flux derived from the entropy generation rate requires that the thermal conductivity be inversely proportional to the square of the temperature; however, there is no known material whose thermal conductivity obeys this relation, so there is no real heat transfer problem where the steady state can be optimized by minimizing the entropy generation rate.

Since the entropy generation theory has limited applications for heat transfer optimization, Guo et al. [18,19] proposed entransy theory. The entransy dissipation rate extremum recovers the steady-state governing differential equation through its spatial variation [14,20]. Thus, it can be used to construct the least action principle (variational principle) in this case, which provides as a simple alternative formulation of the differential equations and can be used to develop the efficient numerical methods. Importantly, the least action principle can be further employed to optimize a problem for given constraints. As for heat transfer optimization in steady state, an entransy dissipation extremum

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Nomenclature

L	plate length
R_t	embedded tube radius
D	pore axis distance
T	temperature
t_{ch}	characteristic time of TES unit
c_V	specific heat
k	thermal conductivity
T	temperature
h	heat transfer coefficient
T_0	initial temperature
T_f	bulk fluid temperature
G	total entropy generation rate

Greek symbols

ρ	mass density

Subscripts

ch	characteristic
ω	frequency domain

principle can be straightforwardly used for some relatively simple problems such volume-point problem [20], while for some complicated problems such heat exchanger networks [21], the entransy dissipation could be used to simplify the constrains and thus facilitate the optimization process. Particularly, several examples about heat transfer optimization based on the entransy theory were also given in a review paper we have cited [19]. For instance, in the volume-point problem, the minimum average temperature rise should correspond to the total entransy dissipation rate extremum; besides, the total entransy dissipation rate were used to derive the optimal fluid flow and temperature fields for laminar convective heat transfer in a circular tube.

As we stated above, the entransy theory in the present form has been successfully utilized for steady-state heat transport process; however, it is still an open question whether this theory could be directly utilized in transient cases. In order to answer this question, we need firstly examine whether the entransy dissipation rate can derive the transient heat conduction equation, and then investigate the applicability of the entransy dissipation rate for transient heat transfer optimization problems.

2. Least action principle for transient heat conduction

The total entransy dissipation rate over a volume V is calculated as

$$G = \int_V q \cdot (-\nabla T) dV = \int_V k(\nabla T)^2 \, dV, \tag{1}$$

in which q is the heat flux, ∇T is the temperature gradient, and k is the thermal conductivity. It is an integral quantity merely over the space domain, and does not involve the influence of the time evolution. In a previous work [14], we have demonstrated the total entransy dissipation rate can be used to construct the least action principle and give the steady-state heat conduction governing equation. However, this conclusion becomes invalid in transient cases. In Finlayson's book [17], the least action principles were summarized for various transport processes including heat conduction; it was found that for a specific transport process, the variational function in the steady state could be different from that in the transient state. Some modifications such as convolution integral [22] and Laplace transform [23,24] are needed to develop the variational principles for transient transport processes. In order to find a variational function in the time domain and without introducing some new variable, convolution integral is preferred to construct the least action principle for transient heat conduction process. Here, following Gurtin [17,22], we derive a variational principle for transient heat conduction using the convolution integral. The convolution integral of the action for the transient heat conduction equation is given by

$$L_{con} = \int_V (c_V \rho T - 2c_V \rho T_0) * T + k * \nabla T * \nabla T \, dV \tag{2}$$

The variational of L_{con} with respect to T yields

$$\delta L_{con} = 2 \int_V \delta T * [c_V \rho T - c_V \rho T_0 - \nabla \cdot (k * \nabla T)] dV = 0 \tag{3}$$

The term in brackets can be rewritten as,

$$c_V \rho T - c_V \rho T_0 - \nabla \cdot (k * \nabla T) = c_V \rho T - c_V \rho T_0 - \int_0^t \nabla \cdot (k \nabla T) d\tau \tag{4}$$

which is the time integral of the transient heat conduction equation,

$$c_V \rho \frac{\partial T}{\partial t} - \nabla \cdot (k \nabla T) = 0. \tag{5}$$

It is obvious that the entransy dissipation rate, Eq. (1), fails to give the transient heat conduction equation; thus, the entransy dissipation rate also likely becomes inapplicable for transient heat conduction optimization. Nevertheless, when carefully analyzing the variational function of transient heat conduction process, Eq. (2), we find that the term, $\int_V k * \nabla T * \nabla T dV$, can be rewritten as

$$\begin{aligned} \int_V \int_0^t k \nabla T * \nabla T d\tau \, dV &= \int_V \int_0^t q * (-\nabla T) dt \, dV \\ &= \int_V \int_0^t \int_0^\tau q(\tau - \eta) \cdot [-\nabla T(\eta)] d\eta \, d\tau \, dV. \end{aligned} \tag{6}$$

It is the integral of the convolution of heat flux and negative temperature gradient over the time and space domain, which could have a close connection to the entransy dissipation rate. Therefore, we deduce that this quantity may be relevant to the dissipative characteristics of the transient heat conduction process, just like the entransy dissipation rate can reflect the dissipative characteristics in steady-state cases [18]. Importantly, the influence of the time evolution of transient transport process can be taken into account by the convolution integral in this quantity. In addition, a transient heat conduction process includes both the dissipating and non-dissipating processes, and thus its variational function could include both the dissipating and non-dissipating terms. Since $(c_V \rho T - 2c_V \rho T_0) * T$ in L_{con} does not involve the driving force or the flux [14], it could be the non-dissipating term.

3. Transient heat conduction optimization problem

To examine the applicability of the entransy dissipation rate for transient heat transfer optimization, a simple transient heat conduction optimization problem is studied in this section. The geometry shown in Fig. 1 is a rectangular plate with an initial uniform temperature of $T_0 = 300$ K at $t = 0$. At $t > 0$, the right