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Heat transfer from moving exponential fins exposed to heat generation

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ABSTRACT

The heat transfer through exponential fins in movement and under the influence of heat generation (absorption) is the fundamental subject of the present paper. Obtaining exact formulas for the thermal features like the distribution of temperature as well as the efficiency of exponential fins is the main target. As a consequence of the current research, combined effects of the Peclet number due to the fin movement, and of the heat generation parameter due to the presence of a heat source on the efficiency of exponential fins versus rectangular fins can be analyzed and compared. Better fin efficiency anticipated for growing type of exponential fins provides practical advantages for the designing engineers. Moreover, simulation of more complex phenomena using modern numerical schemes can be tested by the presented closed-form formulae.

1. Introduction

In order to achieve enhanced heat transfer rates while cooling down the modern technological instruments, engineers look for more convenient and economic means. The extended surfaces or fins are such ideal effective tools during the thermal processing in many engineering based applications, like air conditioning, air-cooled craft engines, and computer equipments, see for instance, the leading books by Kraus et al. [1] and Kalpakjian [2]. Recent research in the active field exhibits that moving fins of different profile sections [3] are significant as well as when they are exposed to a heat source [4]. The present work is hence concerned with the exponential type fins and their potential effects on the transport of thermal properties and the fin efficiency when such fins are set into motion together with a mounted heat source, generating or absorbing heat over the surfaces.

Due to the growing practical necessities, the recent literature witnessed many publications dealing with finned heat transfer over various extended surfaces. Sharqawy and Zubair [5] obtained analytical solutions to examine the efficiency of straight fins and how they can be optimized subject to mass transfer. By devising a mathematical technique to compare with the recently often used Homotopy Analysis Method, Rashidi and Erfani [6] studied the nonlinear effects possessing variable thermal features involved in fin problems. Thermo-geometric fin parameter and thermal conductivity parameter impacts over the fin efficiency of convective straight fins were simulated in the numerical work of Joneidi et al. [7]. Apart from the well-known fin shapes, exponential kind fins also constitute a major role in heat transfer enhancement. Indeed, the analytical work fulfilled by Turkyilmazoglu [8] indicated that better fin efficiency is obtainable via fins of exponential type. The temperature in a porous fin was numerically explored by Hatami et al. [9] over porous fin materials Si3N4 and AL under the influence of a linear heat generation. It was concluded that the AL fin substrate leads to more temperature than Si3N4 element. In a series of papers, Hatami and Ganji [10–12] investigated the applications of porous fins to micro channel heat sink, to refrigeration efficiency and to ceramic-based materials. It was found in Hatami et al. [13] that the Lewis number must be high enough to increase refrigeration efficiency by means of fully wet semi-spherical porous fins. Wet loading conditions were demonstrated to perform better in terms of heat transfer over exponential fin cross sections in the work of Turkyilmazoglu [14]. In the very recent study by Kundu and Lee [15], an analytical analysis was presented to determine a minimum envelop shape of porous fins for a constraint heat transfer rate over the fin surface. It was fascinatingly concluded that the optimum fin shape is inclined to have trapezoidal or exponential geometry instead of a triangular or rectangular one. The helical fin on hydrothermal analysis in a water to air heat exchanger was recently examined both numerically and experimentally by Sheikholeslami and Ganji [16]. Better thermal performance was shown to occur in higher open area ratio.

Along with the fins attached to a stationary surface as in the aforementioned literature, there is also a renewed recent interest in the study of fins when they are set into motion. Such movement of fins find applications, for instance in the manufacturing processes, refer to Fisher [17] and Jiji [18]. Therefore, the moving fins and their thermal activities were thoroughly investigated by

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Nomenclature

Roman symbols

(a_i, b_i, c)	dummy variables
${}_pF_q$	generalized hypergeometric function
h	convective heat coefficient
k	thermal conductivity
L	fin length
N	Biot number
Pe	Peclet number
q	internal heat generation (absorption)
Q	internal heat generation (absorption) parameter
T	dimensional fin temperature
T_b	base temperature
T_a	ambient temperature
U	fin speed
x	dimensional distance
X	dimensionless distance

Greek symbols

α^*	fin shape
α	fin shape parameter
η	fin efficiency parameter
γ	Euler gamma number
θ	dimensionless fin temperature
$\theta(0)$	tip temperature
θ_a	parameter for sink temperature
θ_i	constants

several researchers. Aziz and Lopez [19] numerically solved the moving fin heat transfer equations and they showed that a moving fin affects the convection and radiation in a different manner than a stationary correspondent. Similar conclusion was made in the article by Saedodin et al. [20] taking into account of multiple nonlinearities. An inverse problem was considered by Singla and Das [21] to estimate the speed of moving fin under the given thermal conditions. A mechanism of stretching/shrinking of rectangular fins was introduced by Turkyilmazoglu [22], according to which more heat transfer enhancement and efficiency take place for the shrinking case. Sun et al. [23] proposed a spectral collocation method to compute the convective-radiative heat transfer character of a moving rod, present in the thermal processing of continuous casting and rolling. Moving fins of complex cross-sections were numerically simulated via a spectral collocation method by Sun and Xu [24]. In a recent article by Ma et al. [25], heat transfer in moving irregular porous fins was calculated using a spectral element method.

In the light of the rich collection of aforementioned recent literature, the main motivation for the current work is as follows. In contrast to the rectangular straight fins as often used in real-life applications, particularly the recent researches by Turkyilmazoglu [8,14], by Hatamia and Ganji [12] and by Kundu and Lee [15] clearly revealed that exponentially shaped fins may take the place of the traditional rectangular ones at the expense of better fin efficiency and heat transfer rate desired in industrial applications. Taking this view into account, it is intended here to examine the combined effects of fin movement and heat generation (absorption) by deriving exact analytical solutions for the associated governing heat equation within the assumptions of constant thermo-physical properties and constant heat transfer coefficient. Temperature field, fin efficiency and heat transfer can be easily studied and compared for different exponential fin shapes from the presented closed-form equations. They are also important for checking the accuracy of semi-analytical or full numerical schemes employed to simulate the heat transfer phenomena under consideration.

2. Problem formulation

As some of simple configurations are illustrated in Fig. 1, prime interest here is the heat transfer phenomenon taking place along one-dimensional fins of straight rectangular and exponential profiles with length L.

Since the present target is to get exact solutions for the considered problem, all the physical properties are taken as constants. The fin is assumed to move horizontally with a constant speed U along the axial x coordinate. Well-insulated condition is adopted at the fin tip. The local shape of exponential fin is given by

$$f(x) = e^{2\alpha^* x}, \tag{1}$$

where α^* is named here as the dimensional fin shape parameter [8,14] (the thickness constant is presumed to be unity without loss of generality). It is noted that $\alpha^* = 0$ represents the classical rectangular type fin.

The physical problem is mathematically formulated by pursuing an energy balance resulting in governing energy equation, see for instance the articles [21,22]. Further employing the non-dimensional counterparts via

$$X = \frac{x}{L}, \quad \alpha = \alpha^* L, \quad \theta = \frac{T}{T_b}, \quad \theta_a = \frac{T_a}{T_b},$$
$$N^2 = \frac{hL^2}{k}, \quad Pe = \frac{UL}{\lambda}, \quad Q = \frac{qL^2}{kT_b}, \tag{2}$$

the dimensionless temperature distribution over exponential fins is to be determined from

$$\frac{d}{dX}\left(e^{2\alpha X}\frac{d\theta}{dX}\right) - N^2(\theta - \theta_a) + Pe\, e^{2\alpha X}\frac{d\theta}{dX} + Qe^{2\alpha X} = 0,$$
$$\theta'(0) = 0, \quad \theta(1) = 1. \tag{3}$$

Having integrated the system in (3), we define the fin efficiency η related to the exponential fins (1) by dividing the total heat transfer and maximum heat transfer [14] to get the physically significant parameter

$$\eta = \frac{e^{2\alpha}\theta'(1)}{N^2(1-\theta_a)}. \tag{4}$$

$\theta(0)$, essentially the unknown fin tip temperature is yet other important parameter from the engineering point of view. These parameters as well as the temperature field over exponential fins are seen to be governed by the Biot number N, the sink temperature parameter θ_a, the Peclet number Pe, the heat source parameter Q and the fin shape parameter α. In particular, the heat transfer under the influence of Pe and Q will constitute the new contribution of the present paper.

3. Results and discussions

The rare exact solutions as presented in the book by Kraus et al. [1] for other fin shapes will be enriched by the analytical temperature fields computed from (3) related to exponential fin shapes. Pursuing the research in [8,14], the energy Eq. (3) is solved exactly, having a lengthy expression for the solution in general case, involving advanced mathematics functions, like the Bessel, the modified Bessel, the hypergeometric, the modified hypergeometric, the Gamma and the MeijerG. However, for the special circumstance