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On the role of axial wall conduction in mini/micro counterflow heat exchangers

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ABSTRACT

Understanding the role of multi-dimensional conjugate heat transfer on the thermal efficiency of mini/micro counterflow heat exchangers is critical for the design of high efficient equipment. This topic is addressed here by considering a simple model with exact solution: the laminar counterflow parallel-plate heat exchanger. Using as starting point the eigenfunction series solution recently obtained by the authors, a thorough parametric study is carried out to investigate the role of the two dimensionless parameters involved in multi-dimensional wall conduction: the dimensionless wall thickness, Δ_w, and the dimensionless wall thermal resistance, κ_w^{-1}. The analytical eigencondition is first presented and discussed, and the associated eigenvalue spectrum is analyzed using contour plots of the lowest-order eigenvalues in the $(\Delta_w, \kappa_w^{-1})$ plane. The complex task of determining the eigenvalues numerically is largely facilitated by approximate expressions obtained from the asymptotic analysis of the singularities that appear in the eigencondition. The fast evaluation of the eigenvalues makes it possible to obtain contour plots of the heat exchanger effectiveness in the $(\Delta_w, \kappa_w^{-1})$ plane, which exhibit distinguished regimes corresponding to limiting cases with and without axial and transverse wall conduction effects, with smooth transitions occurring for moderately small values of Δ_w and κ_w^{-1}. The analysis provides conditions for neglecting axial and transverse wall conduction, and shows that an optimum wall conductivity always exists in heat exchangers with sufficiently thin walls.

1. Introduction

The growing interest in the design and fabrication of mini/micro heat exchangers, driven by the ever-increasing miniaturization of micro-electro-mechanical Systems (MEMS), is evidenced by the large number of review articles published in the last years [1–3]. Although the performance of mini/micro heat exchangers could qualify as satisfactory, the relatively large thickness of the partition walls compared to the hydraulic diameter of the channels is known to enhance the role of axial wall conduction, which reduces the effectiveness of counterflow heat exchangers [4] to that of co-flow systems in the limit of large axial conduction [5,6]. In addition to the enlarged wall thickness, the thermal conductivity of the wall is another physical parameter that affects axial wall conduction, and therefore the effectiveness of counterflow heat exchangers [7,8]. Emerging technologies and cost saving strategies have led to the use of a great variety of materials with different thermal conductivities [9–11]. As a result, a better understanding of the impact of multi-dimensional fluid-wall conjugated heat transfer on the thermal performance of mini/micro counterflow heat exchangers is still needed for the development of high efficient equipment.

The first study on the influence of axial wall conduction in balanced and unbalanced counterflow parallel-plate heat exchangers was carried out fifty years ago by Kroeger [12], who demonstrated that the effectiveness is larger for unbalanced heat exchangers, especially for small wall thermal conductivities. His results confirmed previous studies carried out for balanced heat exchangers [13]. In another seminal paper, Mori et al. [14] considered a similar configuration and compared their model predictions with those provided by the classical ε-NTU method. They concluded that when axial wall conduction becomes important the ε-NTU method does not give appropriate guidelines for the design of counterflow heat exchangers.

Pagliarini & Barozzi [15] investigated analytically and numerically the role of axial wall conduction on the thermal performance of laminar counterflow double-pipe heat exchangers. With this aim, they combined first- and second-law analyses, calculating the local entropy production rate both in the fluids and in the wall. They concluded that when axial wall conduction

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Nomenclature

A	expansion coefficient
a_i	channel half-width of fluid i
C_n	expansion coefficient corresponding to the n-th eigenfunction
c_i	specific heat of fluid i
f_n	n-th eigenfunction for thermally thin walls without axial wall conduction
h_n	n-th eigenfunction for the general problem
h_i	heat-transfer coefficient of fluid i
k	dimensionless parameter, $a_1 k_2/(a_2 k_1)$
k_i	thermal conductivity of fluid i
k_w	thermal conductivity of the wall
L	heat-exchanger length
M	Whittaker function, $M_{\kappa,\mu}(z)$
m	dimensionless parameter, $a_2\mathrm{Pe}_2/(a_1\mathrm{Pe}_1)$
Nu_i	Nusselt number of fluid $i, h_i(4a_i)/k_i$
Pe_i	Peclet number of fluid $i, 2a_iV_i/\alpha_i$
T	temperature
V_i	average flow velocity of fluid i
W	Whittaker function, $W_{\kappa,\mu}(z)$
X	longitudinal distance from the inlet of fluid 1
Y_i	transverse distance from channel i symmetry plane
Y_w	transverse distance from the wall symmetry plane
y_i	dimensionless transverse coordinate, Y_i/a_i
y_w	dimensionless transverse coordinate, $Y_\mathrm{w}/\delta_\mathrm{w}$

Greek letters

α_i	thermal diffusivity of fluid $i, k_i/(\rho_i c_i)$
Δ_w	dimensionless parameter, $\delta_\mathrm{w}/(a_1\mathrm{Pe}_1)$
δ_w	wall thickness
ε	heat exchanger effectiveness
Γ	Gamma function, $\Gamma(z)$
κ	first argument of Whittaker functions
κ_w	dimensionless parameter, $k_\mathrm{w}a_1/(\delta_\mathrm{w}k_1)$
λ_n	n-th eigenvalue
μ	second argument of Whittaker functions
ν_w	dimensionless average wall-axial heat flux
ν_i	dimensionless interfacial wall-normal heat flux of fluid i
ν_i	kinematic viscosity of fluid i
ρ_i	density of fluid i
θ_i	normalized temperature, $(T_i - T_{1,\mathrm{in}})/(T_{2,\mathrm{in}} - T_{1,\mathrm{in}})$
ξ	dimensionless longitudinal coordinate, $X/(a_1\mathrm{Pe}_1)$
ξ_L	dimensionless parameter, $L/(a_1\mathrm{Pe}_1)$

Subscripts

i	subscript used indistinctly for fluids 1 and 2
in	inlet
L	heat-exchanger length
m	bulk, or mixing-cup, temperature
n	n-th eigenvalue/eigenfunction
out	outlet
w	heat-exchanging wall

becomes significant, the local heat transfer rates at the fluid-wall interfaces are no longer coupled, and tend to reach an asymptotic value far from the inlets. The Nusselt numbers behave similarly, being lower bounded by the classical value for constant wall temperature, because the wall temperature becomes longitudinally uniform in the presence of strong axial wall conduction. By contrast, for negligible axial wall conduction the heat exchanger effectiveness increases monotonically with the thermal conductivity of the partition wall. When axial wall conduction is important, the overall entropy production rate exhibits a minimum that goes hand in hand with a maximum in the effectiveness at an optimum intermediate wall conductivity.

Bier et al. [7] observed experimentally that, under certain conditions, micro heat exchangers made of stainless steel (lower conductivity) present higher effectiveness than micro heat exchangers made of copper (higher conductivity), concluding that axial wall conduction is an important factor determining system performance. Stief et al. [8] confirmed that reducing the wall conductivity may lead to higher efficiencies in micro heat exchangers due to the detrimental effect of axial heat conduction in the channel walls. Very high wall conductivities offset the axial temperature gradients within the wall to such an extent that the heat transfer efficiency of any micro heat exchanger, regardless of the flow direction specified, always approximates the lower heat transfer efficiency of co-current flow heat exchangers. On the other hand, as the wall conductivity tends to zero the insulating effect associated with the increased wall thermal resistance makes the efficiency to approach zero as well, so that an optimum intermediate wall conductivity must exist.

Maranzana et al. [16] studied the influence of axial wall conduction on the performance of parallel-plate counterflow micro heat exchangers. Results computed ignoring axial wall conduction (1D wall model) and including both transverse and axial wall conduction effects (2D wall model) were obtained and compared. It was shown that neglecting axial conduction overestimates heat transfer, as intense axial wall conduction turns the counterflow exchanger into a mixer (a system where the outlet temperatures of both fluids tend to be equal), the consequence of axial conduction being to lower the heat exchanger effectiveness. As a result, they concluded that an optimum conductivity must exist for the wall, and that disregarding the effect of axial conduction in the wall can lead to a large bias in the experimental estimation of heat transfer coefficients, particularly for small Reynolds numbers. As main outcome of the analysis, the so-called axial conduction number M was introduced by comparing axial heat transfer by conduction in the wall to convective heat transfer in the flow, concluding that the effect of axial wall conduction could be neglected for $M < 0.01$.

Moreno et al. [17] employed a simple 1D model to investigate the steady-state heat transfer between two fluids in a microscale balanced heat exchanger with adiabatic and isothermal end-wall boundary conditions. Results obtained for the case of isothermal packaging demonstrated that the effectiveness is a weak function of the axial conduction number. As the thermal conductivity of the wall grows, conduction heat losses increase correspondingly, but the effectiveness remains high because the wall temperature is fixed by the external conditions. These results demonstrated the benefit of employing intermediate thermal conductivity wall materials to maintain rapid heat transfer while minimizing conductive heat losses via packaging. Results obtained for the case of adiabatic packaging demonstrated that heat exchanger effectiveness is significantly reduced with increasing wall thermal conductivity, when the system approaches the limit of thermal equilibrium between the two fluids and the wall, equivalent to co-current operation. By contrast, low thermal conductivity materials allow maintenance of substantial thermal gradients along the solid-phase axial length by preventing thermal equilibration.