International Journal of Heat and Mass Transfer 116 (2018) 969–976

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Lattice Boltzmann simulation of shear viscosity of suspensions containing porous particles

A. Xu, L. Shi, T.S. Zhao *

HKUST Energy Institute, Department of Mechanical and Aerospace Engineering, The Hong Kong University of Science and Technology, Hong Kong, China

ARTICLE INFO

Article history:
Received 1 July 2017
Received in revised form 11 September 2017
Accepted 17 September 2017
Available online 28 September 2017

Keywords:
Relative viscosity
Porous particle
Particle flow
Lattice Boltzmann method

ABSTRACT

We present three-dimensional lattice Boltzmann simulations of dilute suspensions containing porous particles. The fluid flow around and inside a porous particle is described by the volume-averaged macroscopic equations in terms of intrinsic phase average. The energy dissipation of the suspended particle in a Couette flow is calculated to obtain the relative viscosity of the suspension. Results show that the relative viscosity of the suspension increases linearly with the particle volume fraction. A correlation equation is obtained for the intrinsic viscosity as a function of Darcy number. It is found that when the suspension is at the inertial flow regime, its intrinsic viscosity increases linearly with Reynolds number, and the increasing rate depends on Darcy number.

1. Introduction

Suspended solid particles in a fluid are ubiquitous in nature and have wide applications in industry. Determining the rheology of a particle suspension is important to control industrial flow processes accurately [1]. The rheology of a suspension is usually characterized by the relative viscosity η_r, which is the ratio between effective viscosity of a suspension and viscosity of the base fluid. The first theoretical study to determine the relative viscosity may date back to 1906, when Einstein derived the mathematical expression $\eta_r = 1 + \langle\bar{\eta}\rangle\phi$ for a dilute suspension containing spherical particles [2]. Here, ϕ is solid particle volume fraction, $\langle\bar{\eta}\rangle$ is intrinsic viscosity and takes the value of $\langle\bar{\eta}\rangle = 2.5$. Afterwards, to study a semi-dilute suspension that has higher particle volume fraction, the relationship $\eta_r = 1 + \langle\bar{\eta}\rangle\phi + \langle\bar{\eta}\rangle_1\phi^2 + \cdots$ was proposed in which higher order terms of ϕ were included [3]. For a concentrated suspension that has even higher particle volume fraction of $\phi > 25\%$, the relationship $\eta_r = (1 - \phi/\phi_m)^{-B\phi_m}$ was proposed so that the relative viscosity would approach infinite when the particle volume fraction is approaching the densest possible packing fraction ϕ_m [4]. In addition to suspensions of spherical particles, research attentions are also drawn on non-spherical particles. Examples include Jeffery's analytical solution of relative viscosity for suspensions of ellipsoidal particles that are prolate or oblate [5]. Recently, with the tremendous increase in computational capability, first-principle-based modeling techniques that are capable of handling complex moving boundary problems are also adopted as an alternative strategy to determine the rheology of suspensions [6–11]. The basic idea behind these numerical approaches is to obtain the energy dissipation of the suspended particle in a Couette flow through direct numerical simulation of the flow field. For example, Lishchuk et al. [7] calculated the shear viscosity of suspensions of spherical particles and reproduced both the Einstein's relation for low particle volume fraction and Krieger-Dougherty's relation for high particle volume fraction. Moreover, these numerical approaches do not restrict the flow at low Reynolds number (*Re*), which allows to investigate the dependence of intrinsic viscosity on *Re*. For example, Kulkarni and Morris [8] reported the relative viscosity of suspension of spherical particles at $0.01 \leqslant Re < 5$. They showed that inertia can increase the particle contribution to the effective viscosity of the suspension. Huang et al. [11] found that for dilute prolate and oblate spheroidal suspensions, the intrinsic viscosity changes linearly with *Re* at low-*Re* regime and nonlinearly at high-*Re* regime.

The above studies focused on solid particles that are impermeable to fluid, while in real-world applications, porous particles that are permeable to fluids are also frequently encountered, such as core-shell like particles, catalyst clusters, encapsulated drugs, and so on [12–14]. Efforts have been devoted to study the effect of permeability of the porous particle on the flow pattern and the particle-fluid interactions [15,16]. For example, Masoud et al. [15] theoretically investigated the dynamics of porous elliptical particle in shear flow. They concluded that although the flow field inside and around the particle significantly depends on the permeability of the porous particle, the permeability has little effect on

* Corresponding author.
E-mail address: metzhao@ust.hk (T.S. Zhao).

https://doi.org/10.1016/j.ijheatmasstransfer.2017.09.060
0017-9310/

the rotational behavior of the porous particle. Later, Li et al. [16] numerically investigated the dynamics of porous circular particle in shear flow. They found Masoud et al.'s conclusion is only validated when the fluid inertia is negligible or the confinement effect of the bounding walls is weak.

Although progress has been made on the research of suspensions containing porous particles, to the best of our knowledge, there is no investigation on its rheology with the aid of first-principle-based modeling techniques. In this work, we present three-dimensional lattice Boltzmann (LB) simulation of a dilute suspension containing porous particle to determine the relative viscosity. To describe the fluid flow around and inside a porous particle, the general volume-averaged conservation equations are adopted [16–18]. When the fluid motion is sufficient slow, the Darcy's law [19] and the Brinkman-Debye-Bueche (BDB) equation [20,21] can be recovered; when the flow Reynolds number is finite, the inertial effect on the flow inside and around the porous particle can be incorporated. More importantly, the continuity of both fluid velocity and shear stress at the interface between the porous region and the free flow is ensured through the incorporating of a second-order viscous term in the volume-averaged macroscopic governing equation. Here, the lattice Boltzmann (LB) method is chosen to obtain the numerical solution due to its simplicity, accuracy, and parallelism for simulating complex multiphase flows [22–24].

2. Numerical method

2.1. Volume-averaged macroscopic governing equations

The fluid flow around and inside the porous particle is described by the volume-averaged equations in terms of intrinsic phase average proposed by Wang et al. [18], which is written as

$$\nabla \cdot \langle \mathbf{u}_f \rangle^f = 0 \tag{1}$$

$$\frac{\partial \langle \mathbf{u}_f \rangle^f}{\partial t} + \langle \mathbf{u}_f \rangle^f \cdot \nabla \langle \mathbf{u}_f \rangle^f = -\frac{1}{\rho_f} \nabla \langle p_f \rangle^f + \nu \nabla^2 \langle \mathbf{u}_f \rangle^f + \mathbf{F}_m \tag{2}$$

where ρ_f is the fluid density and ν is the fluid viscosity. $\mathbf{u}_f$ and p_f are the local fluid velocity and pressure, respectively. Here, the intrinsic phase average is defined as $\langle \psi_k \rangle^k = \frac{1}{V_k} \int_{V_k} \psi_k dV$, and the phase average is defined as $\langle \psi_k \rangle = \frac{1}{V} \int_{V_k} \psi_k dV$. V_k denotes the volume of the k-phase within the representative volume V. ψ_k is a quantity associated with the k-phase. The subscript s and f represent solid and fluid phase, respectively. The total body force $\mathbf{F}_m$ in Eq. (2) is given by [18]

$$\mathbf{F}_m = -\frac{\varepsilon \nu}{K} \left(\langle \mathbf{u}_f \rangle^f - \langle \mathbf{u}_s \rangle^s \right) - \frac{\varepsilon^2 F_\varepsilon}{\sqrt{K}} \left(\langle \mathbf{u}_f \rangle^f - \langle \mathbf{u}_s \rangle^s \right) \left| \langle \mathbf{u}_f \rangle^f - \langle \mathbf{u}_s \rangle^s \right| \tag{3}$$

where ε is the porosity of the porous particle. The first and the second terms on the right-hand side of Eq. (3) are the linear Darcy drag and nonlinear Forchheimer drag due to the existence of porous medium, respectively. Because the porous particle moves as a rigid-body, its translational and rotational velocities does not change after taking the intrinsic phase averaging. Then, the intrinsic phase-average velocity of the solid particle $\langle \mathbf{u}_s \rangle^s$ is calculated as $\langle \mathbf{u}_s \rangle^s = \mathbf{u}_s$. The permeability K quantifies the ability of the porous particle to transmit fluids. F_ε is the geometric function and it is given as $F_\varepsilon = 1.75/\sqrt{150\varepsilon^3}$ following Ergun's correlation. The porous structure inside the particle is described by the permeability K and the porosity ε, which can be correlated via the relation $K = \varepsilon^3 d_p^2 / \left[150(1-\varepsilon)^2\right]$ to simplify the problem. Here, d_p is the characteristic diameter of filling grains within the porous particle. In the limit of $\varepsilon \to 0$, the porous particle would reduce to an impermeable particle; while in the limit of $\varepsilon \to 1$, the regime occupied by the porous particle would be filled with fluid. The dimensionless numbers characterize the system are the particle Reynolds number (Re_p) and Darcy number (Da), which are defined as

$$Re_p = \frac{\Gamma D^2}{\nu}, \quad Da = \frac{K}{D^2} \tag{4}$$

where Γ is the shear rate, and D is the diameter of the porous particle.

2.2. Lattice Boltzmann model for volume-averaged equations

In LB method, to solve Eqs. (1) and (2), the evolution equation of density distribution function can be written as

$$f_i(\mathbf{x} + \mathbf{e}_i \delta_t, t + \delta_t) - f_i(\mathbf{x}, t) = -\left[f_i(\mathbf{x}, t) - f_i^{(\mathrm{eq})}(\mathbf{x}, t) \right] + \delta_t F_i, \quad (i = 0, 1, \ldots, 18) \tag{5}$$

where f_i is the density distribution function, t is the time, δ_t is the time step, $\mathbf{x}$ is the fluid parcel position. $\mathbf{e}_i$ is the discrete velocity along the ith direction, and is given as

$$\begin{aligned} &[\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4, \mathbf{e}_5, \mathbf{e}_6, \mathbf{e}_7, \mathbf{e}_8, \mathbf{e}_9, \mathbf{e}_{10}, \mathbf{e}_{11}, \mathbf{e}_{12}, \mathbf{e}_{13}, \mathbf{e}_{14}, \mathbf{e}_{15}, \mathbf{e}_{16}, \mathbf{e}_{17}, \mathbf{e}_{18}] \\ &= c \begin{bmatrix} 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \end{bmatrix} \end{aligned} \tag{6}$$

where $c = \delta_x/\delta_t$ is lattice constant, and $c = \delta_x = \delta_t = 1$ is adopted in this work. The equilibrium particle distribution function is

$$f_i^{(\mathrm{eq})}(\mathbf{x}, t) = \rho_f \omega_i \left[1 + \frac{\mathbf{e}_i \cdot \langle \mathbf{u}_f \rangle^f}{c_s^2} + \frac{\left(\mathbf{e}_i \cdot \langle \mathbf{u}_f \rangle^f \right)^2}{2c_s^4} - \frac{|\langle \mathbf{u}_f \rangle^f|^2}{2c_s^2} \right], \quad (i = 0, 1, \ldots, 18) \tag{7}$$

where the weights are $\omega_0 = 1/3$, $\omega_{1-6} = 1/18$, $\omega_{7-18} = 1/36$, and $c_s^2 = 1/3c^2$ is the lattice sound speed. The forcing term is given by [16]

$$F_i = \rho_f \omega_i \left(1 - \frac{1}{2\tau} \right) \left[\frac{\mathbf{e}_i \cdot \mathbf{F}_m}{c_s^2} + \frac{\mathbf{e}_i \cdot \langle \mathbf{u}_f \rangle^f}{c_s^4} (\mathbf{e}_i \cdot \mathbf{F}_m) - \frac{\langle \mathbf{u}_f \rangle^f \cdot \mathbf{F}_m}{c_s^2} \right], \quad (i = 0, 1, \ldots, 18) \tag{8}$$

The macroscopic density ρ is calculated as

$$\rho = \sum_{i=0}^{18} f_i \tag{9}$$

The macroscopic velocity $\langle \mathbf{u}_f \rangle^f$ is calculated as