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Constructal design applied to elliptic tubes in convective heat transfer cross-flow of viscoplastic fluids

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ABSTRACT

The objective of this work was to obtain geometries that maximize the heat transfer and minimize pressure drop for viscoplastic fluids in cross flow around elliptical section tubes. The Constructal Design associated with exhaustive search was employed to obtain the ellipse aspect ratios that maximize the Nusselt number and to minimize dimensionless pressure drop, for fixed parameters such as ellipse area, Reynolds ($Re = 1$), Prandtl ($Pr = 1$) and Herschel-Bulkley ($HB = 1$) numbers. The viscoplastic fluid behavior was modeled using the Herschel-Bulkley constitutive equation. The power-law index, n, was a parameter to predict fluid shear thinning. The system of differential equations for flow and heat transfer was solved numerically by the finite volume method. The aspect ratio to maximize $\overline{Nu}$, $r_{q,opt}$, and the one to minimize $\Delta\tilde{p}$, $r_{p,opt}$, were searched for different values of n, ranging from 0.4 to 1. The $r_{q,opt}$ was found to be very close to 1 for all n's, which corresponds to the tube of circular cross section. Low aspect ratio tubes lead to the formation of greater unyielded zones as compared to high aspect ratio tubes. This may be a key to understand the increase in heat transfer from the tube as the aspect ratio increases to one. The aspect ratio that led to the lowest pressure drop was not the same for all fluids, contrarily to the behavior noticed for thermal purpose. It was noticed that as n decreases, the $r_{p,opt}$, also decreases, meaning that to reduce pressure drop for more shear thinning fluids the tube must be slender. This may lead to the conclusion that elliptic tubes might be a good alternative in applications of heat transfer with mild viscoplastic fluids (low HB) only when there is the need to reduce pressure drop or pumping power.

1. Introduction

External flows of Newtonian and non-Newtonian fluids across cylinders of circular and non-circular cross-sections are the focus of many researches and investigations. There is a great amount of literature dedicated to unconfined infinitely long cylinders subjected to cross-flow of Newtonian fluids, and some classical papers and books dedicated to this subject e.g. [43,40,41]. Even in the case of Newtonian fluids, there are questions that still remain open and give subject for present research e.g. [42].

In the case of non-Newtonian fluids, flows over cylinders and blunt bodies find applications in many engineering problems, such as the design of heat exchangers and the flow through chemical reactors and porous media. Shear-thinning and viscoplasticity are two common non-Newtonian effects in fluids of industrial interest, such as paints, sludges, muds, foods and molten polymers, as well in biological fluids such as blood and synovial fluid. Shear thinning is the effect of viscosity decrease with shear rate increase. The fluid thins due to shear, becoming less viscous in the region adjacent to a wall. Shear thickening is the contrary, the fluid presents increase in viscosity as the shear rate increases. Viscoplasticity is the effect of behaving either as a solid or as a fluid, depending on the level of shear stress. Viscoplastic fluids flow only above a minimum shear stress, named yield stress. Below the yield stress, a viscoplastic material behaves as a solid, or as an extremely viscous fluid. Above the yield stress, it flows as a viscous fluid. There are many works on flows and heat transfer of non-Newtonian fluids around cylinders, where the roles of shear thinning, shear thickening and viscoplasticity were investigated e.g. [10,9,32,12,36]. Nirmalkar et al. [24] presented a definitive work on convection heat transfer from a cylinder to viscoplastic fluids. Using computational fluid dynamics (CFD), they investigated the effect of classical dimensionless parameters, Reynolds (Re) and Prandtl (Pr) numbers, and the effect

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Nomenclature

A	elliptic tube cross-sectional area
a	horizontal ellipse semi-axis
b	vertical ellipse semi-axis
Bn	Bingham number $(\frac{\tau_0}{KU_\infty/(A^{1/2})})$
c	specific heat
C_D	drag coefficient $(\frac{F_D}{\rho U_\infty^2/2})$
$\mathbf{D}$	strain rate tensor
H	vertical size of computational domain, half of the transverse pitch
$\tilde{H}$	dimensionless vertical size of computational domain $\left(\frac{H}{A^{1/2}}\right)$
HB	Herschel-Bulkley number $\left(\frac{\tau_0}{K[U_\infty/(A^{1/2})]^n}\right)$
k	thermal conductivity
L	horizontal size of computational domain
L_e	ellipse perimeter
n	Power-law index
Nu	Nusselt number $\left(\frac{q_w D}{k(T_w-T_\infty)}\right)$
$\overline{Nu}$	average Nusselt number $\left(\frac{q_w(A^{1/2})}{L_e k(T_w-T_\infty)}\right)$
$\overline{Nu}_{max}$	maximum average Nusselt number
p	pressure
$\tilde{p}$	dimensionless pressure $\left(\frac{p}{\rho U_\infty^2/2}\right)$
$\Delta\tilde{p}$	dimensionless pressure drop
$\Delta\tilde{p}_{min}$	minimum dimensionless pressure drop
Pr	Prandtl number $\left(\frac{cK}{k}\left(\frac{U_\infty}{A^{1/2}}\right)^{n-1}\right)$
q_w	heat transfer rate
q_w	heat transfer rate
r	ellipse aspect ratio (b/a)
Re	Reynolds number $\left(\frac{\rho U_\infty^{2-n}(A^{1/2})^n}{K}\right)$
$r_{p,opt}$	optimal aspect ratio for pressure drop
$r_{q,opt}$	optimal aspect ratio for heat transfer
T	temperature
T_∞	temperature at free stream
T_w	temperature at tube wall
$\mathbf{u}$	velocity vector
$\tilde{\mathbf{u}}$	dimensionless velocity $\left(\frac{\mathbf{u}}{U_\infty}\right)$
U_∞	velocity at free stream

Greek symbols

α	thermal diffusivity
$\dot{\gamma}_c$	characteristic strain rate
$\dot{\gamma}$	strain rate
η	viscosity function
θ	angle over the circular tube wall
$\tilde{\theta}$	dimensionless temperature $\left(\frac{T-T_\infty}{T_w-T_\infty}\right)$
ρ	mass density
τ	extra stress tensor

of the Bingham number on heat transfer, using the Nusselt number (Nu). The Bingham number, Bn, is supposed to be a dimensionless quantification of the yield stress [35,37]. Besides observing that the increase of Re or Pr led to an augmentation of heat transfer rate, Nirmalkar et al. [24] showed that the increase in Bn also resulted in higher Nu. This conclusion was contrary to the expectations, due to the formation of unyielded fluid regions, where the material actually does not flow and convection is impaired.

Concerning flows of non-Newtonian fluids across cylinders with non-circular profiles, such as elliptic cross-sections, there are studies focused on obtaining drag coefficients and Nusselt numbers for different non-Newtonian fluids. For example, Putz and Frigaard [30] presented an important work using the Augmented Lagrangian method to obtain numerical solutions for the flow of viscoplastic fluids. In this study, the accuracy in the detection of unyielded zones using this method was one of their main contributions. Rao et al. [31] performed numerical simulations of laminar flows of power-law fluids over elliptic tubes. They investigated the effect of the flow index and the ellipse aspect ratio over the critical Reynolds number. Chabbra [14] presented a wide review on flow of non-Newtonian fluids over cylinders of circular and non-circular cross-sections, with focus on the transition from steady to periodic regimes. They concluded that the rheological characteristics and the shape of the bluff body, its orientation with regard to the mean direction of flow, its extent and type of confinement strongly influence the critical Reynolds number. Additional important work in the view of enhanced heat transfer and enhanced thermal performance has also been recently published by Hajmohammadi [16–19].

The Constructal Law was stated by Professor Adrian Bejan in 1997 and it states that "for a finite-size system to persist in time to live), it must evolve freely in such a way that it provides easier access to the imposed currents that flow through it" [3–5]. Construtal Design happens spontaneously in nature [6,4]. In Engineering, Constructal Design may be used as a method of design of equipment with greater energetic efficiency. One of the applications of Constructal Design in Engineering is in the design of flow systems with the objective of maximizing heat transfer on a fixed volume and subjected to any constraints. For example, Bello-Ochende and Bejan [7] proposed to maximize the heat transfer density on crossflows of Newtonian fluids over cylinders kept at a constant temperature and subjected to a constant pressure drop. The configuration was a non-uniform arrangement of cylinders with different diameters. Working with four degrees of freedom, the authors obtained complex geometries with optimal values of heat transfer density. Kim et al. [22] studied similar problems including the effect of natural convection. Bello-Ochende et al. [8] and Page et al. [27] employed Constructal Design to obtain optimal configurations considering rotating cylinders of different diameters subjected to forced and natural convection, respectively. These works have employed Constructal Design in the evaluation of geometry subjected to external flows of Newtonian fluids. Important works have also been published recently in the Constructal realm, e.g. Bejan [2], Gonzalez et al. [15], Hajmohammadi [20], Hajmohammadi et al. [21], Ziaei et al. [33], Wattez and Lorente [39]. However, the analysis of non-Newtonian flows using Constructal Design has never been done before.

In the present work, we relied on Constructal design associated to the exhaustive search to study flow and heat transfer of viscoplastic fluids (modeled using Herschel-Bulkley equation) over a row of tubes of elliptic cross section. We investigated the effects of the power-law index and the ellipse aspect ratio on pressure drop and heat transfer. The parameters Reynolds (Re), Prandtl (Pr) and Herschel-Bulkley (HB) numbers were fixed. We searched for the best aspect ratios to improve heat transfer and to reduce pressure drop for different values of the power-law index, representing fluids of different shear-thinning features.

The results of this study are the first step towards the Constructal design and optimization of tube bundles of heat exchangers or chemical reactors working with viscoplastic fluids.