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Methodology of surface heat flux estimation for 2D multi-layer mediums

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Surface heat flux is an important parameter in various industrial applications. It is often estimated based on measured temperature by solving inverse heat conduction problems (IHCPs). In the present work, a filter solution to solve 1D single-layer IHCPs is applied to calculate the surface heat flux for 2D multi-layer mediums. An optimal comparison criterion is implemented for 2D IHCPs to optimize the key regularization parameters. Afterward, the 2D filter solution is used for heat flux estimation with thin-film thermocouple (TFTC) and fine thermocouple (FTC) measurements during cryogen spray cooling. The accuracy of the estimated heat fluxes is tested with the measured temperature response to cryogen spray cooling. A small error (maximum value of 1.0740 °C) is observed between the temperature simulated based on estimated heat fluxes and the measured temperature. The maximum heat flux obtained by the 2D filter solution is 13.6% higher than that obtained by 1D method for TFTC measurement. This finding indicates that lateral heat transfer cannot be disregarded, especially when the heat conductivity coefficient of the material is large.

1. Introduction

The heat flux characterizes the heat transfer capacity per unit area and is a significant index to evaluate the heat transfer performance of devices and facilities. Accurate estimation of surface heat flux profiles is important in various industrial applications, such as thermal protection of space shuttles [1], thermal management of electronic devices [2], metal heat treatment [3], maintenance of boilers [4] and nuclear reactors [5], spray cooling [6], and geophysics [7]. However, direct measurement of surface heat flux is difficult. By contrast, temperature measurement is easier. Thus, indirect estimation of surface heat flux by using surface or internal temperature has elicited much attention. Surface heat flux can generally be estimated by solving inverse heat conduction problems (IHCPs) according to the measured surface or internal temperature. The accuracy of surface heat flux estimation can be validated by comparing a hypothetical surface heat flux and one computed based on the temperature simulated with the hypothetical surface heat flux as a boundary condition.

IHCPs are mathematically ill-posed, and a small error in temperature may significantly affect the accuracy of heat flux estimation [8,9]. To solve this kind of problem, many analytical and numerical techniques have been proposed; these techniques include sequential function specification (SFS) [10], transfer function [11], conjugate gradient (CG) [12,13], singular value decomposition (SVD) [14,15], and Tikhonov regularization (TR) [16–18]. The SFS method proposed by Beck et al. [10] minimizes the effect of random errors by using future temperature data obtained with the least-squares method. However, the SFS method may cause uncertain heat flux fluctuation because of the inherent unstable nature of the algorithm when solving a multi-layer geometry [19]. The transfer function method regards heat flux as the input of a dynamic system and temperature history as the response; it constructs the relationship between the input and output by using Green's function [20,21]. This method is simple, and its algorithm is stable. However, solving for the transfer function is difficult when dealing with a complex geometry. CG and SVD methods involve complicated algorithms and often cause inherent oscillations [22]. TR is usually regarded as the entire time domain method, which requires temperature data on all time steps and calculates the entire heat flux simultaneously [18]. All these methods have their advantages and disadvantages. Most of them are weak in terms of solving IHCPs with a complex geometry, and several (e.g., CG and SVD methods) involve complicated algorithms. Recently, a filter solution based on TR has attracted the interest of many researchers [9,23–27]. This solution was developed by minimizing the sum of the squares of the errors between estimated and measured temperatures with respect to the unknown heat fluxes and stabilized by Tikhonov regularization. This solution is expressed in a digital filter form, which allows for an almost real-time heat flux estimation, and has been applied in heat flux

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Nomenclature

a	thermal diffusion coefficient (m^2/s)
c	specific heat capacity (kJ/kg·K)
f	filter coefficient
$\mathbf{F}$	filter coefficient matrix
H_i	thickness of the *i*th layer (mm)
H_T	depth of sensors (mm)
$\mathbf{H_t}$, $\mathbf{H_s}$	temporal and spatial first order regularization matrix
L	spray distance (mm)
m_f, m_p	number of future and past time steps
n	time step index
R_q	sum of squares of the surface heat flux errors
q	surface heat flux (kW/m^2)
$\mathbf{q}$	surface heat flux matrix
S	sum of squares of the temperature errors
t	time (ms)
t_d	time step (ms)
T	temperature of substrates (°C)
$\mathbf{T}$	temperature matrix
W	width of geometry (mm)
$\mathbf{X}$	sensitivity matrix
Y	experimental temperature (°C)

Greek symbols

α_t, α_s	temporal and spatial regularization parameter
λ	heat conductivity coefficient (kW/m·K)
ρ	density (kg/m^3)
ϕ	excessive temperature (°C)
σ_Y	standard deviation of the random measurement errors (°C)
Δt	spray duration (ms)
Δx	interval between every two thermocouples (mm)
ε	uniform random temperature error (°C)
τ	response time of thermocouple

Subscripts

c	threshold value
i	layer index
j	sensor index
k	surface heat flux index
0	initial value
int	interface value
min	minimum value
max	maximum value
MSE	mean standard error

Superscripts

^	estimated value

measurement using a directional flame thermometer [28]. This method demonstrates superiority when solving IHCPs with a complex geometry, and its algorithm is simple. However, this method can only be used to solve 1D single-layer IHCPs directly. Little work has been conducted to solve multi-dimensional, multi-layer IHCPs.

In this study, cryogen spray cooling (CSC) with a spray duration of several tens of milliseconds was selected as an example to investigate multi-dimensional, multi-layer IHCPs because such transient spray cooling often leads to ultra-fast surface temperature variation and a rapid change in time-dependent surface heat flux. In transient CSC, time-dependent surface heat flux can be estimated by solving IHCPs using the internal or surface temperature history of the solid substrate. Two typical measurements, namely, fine thermocouple (FTC) and thin-film thermocouple (TFTC), are commonly used to monitor internal and surface temperatures. For example, Anguilar et al. [29] utilized the SFS method to evaluate surface heat flux from internal temperature (~45 μm from the upper surface) measured by a type-T FTC (50 μm bead diameter) placed underneath a thin layer of aluminum foil (20 μm). The foil was positioned on the top of epoxy resin to provide rapid heat transfer from cooling cryogen droplets and mechanical support. Zhou et al. [19,30,31] measured time-dependent surface temperature by using a 2-μm type-T TFTC magnetically deposited onto an epoxy resin surface; the method accurately captured the temperature variation during CSC because of its ultra-fast thermal response (~1.2 μs). Afterward, surface heat flux was calculated with Duhamel theory. Although the temperature measured with TFTC is closer to the surface temperature than that measured with other methods, TFTC cannot be used to measure the temperature of metal materials because of electrical conductivity. Moreover, TFTC corrodes and oxidizes easily when it is exposed to high-temperature environments. Therefore, FTC measurement is widely used in many industries because of its reliability and stability. Unlike TFTC measurement with its single-layer geometry, FTC measurement consists of three layers, namely, aluminum, thermal paste, and epoxy resin. For generality, multi-layer IHCPs need to be developed.

Our recent work [19] compared 1D SFS, the transfer function, and the Duhamel theory method for TFTC and FTC measurements. The results indicated that the SFS method can be applied for TFTC and FTC measurements, but a noticeable discrepancy in the maximum surface heat flux was discovered. The transfer function method effectively inhibited noise and was suitable for TFTC and FTC measurements. The Duhamel theory method was insensitive to noise but unsuitable for FTC measurement. The Duhamel theory method was extended to the multi-layer case [19], in which surface heat flux was estimated based on the actual surface temperature calculated directly with traditional Duhamel theory from the measured internal temperature rather than the internal temperature. This method was validated in terms of its accuracy and applicability to TFTC and FTC measurements. We refer to this new method as Duhamel theory multi-layer method in this paper.

Most of the abovementioned algorithms are based on 1D IHCP, which is based on the assumption that the lateral temperature distribution is uniform. In reality, the radial and temporal surface temperature variations during CSC result in significant non-uniformity of the surface heat flux [32,33]. Therefore, lateral heat transfer must be considered. Theoretically, surface heat flux distribution can be evaluated with a 2D IHCP model, especially when the heat conductivity coefficient is large. Therefore, a general 2D multi-layer IHCP needs to be developed. Najafi et al. [8] presented a filter solution for a 2D inverse heat conduction problem. However, the corresponding regularization and filter parameters that influence the accuracy of the estimated heat flux are given directly without any optimization. Also, the solution cannot be used for the evaluation of multi-layer IHCPs, which is essential for FTC measurement. In summary, a general 2D multi-layer IHCP is necessary.

In the current work, a filter solution to solve 1D single-layer IHCPs was applied for a general 2D, multi-sensor, multi-layer surface heat flux estimation problem to consider lateral heat transfer.