International Journal of Heat and Mass Transfer 111 (2017) 1063–1070

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Instability of parallel buoyant flow in a vertical porous layer with an internal heat source

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ARTICLE INFO

Article history:
Received 16 March 2017
Received in revised form 13 April 2017
Accepted 13 April 2017

Keywords:
Porous medium
Buoyant flow
Internal heat source
Linear stability
Eigenvalue problem
Rayleigh number

ABSTRACT

Buoyant flow in a vertical porous layer whose open boundaries are kept at uniform and different temperatures is analysed. The presence of a uniform volumetric heat source alters the conduction profile of the temperature field for the stationary parallel flow. It is shown that this stationary flow becomes unstable when either the temperature difference between the boundaries or the intensity of the volumetric heat source are sufficiently large. The linear instability is investigated through a study of normal mode disturbances. The stability eigenvalue problem is solved numerically by employing the shooting method. The neutral stability curves are obtained and the critical parameters at onset of instability are determined.

1. Introduction

Enhancement of heat transfer across a porous layer saturated by a fluid can be an important aspect for either engineering and geophysics. Engineering applications range from insulation techniques for buildings and, specifically, for breathing walls to chemical engineering and the design of packed bed reactors. Geophysical applications are relative to groundwater dynamics and the characterisation of thermal environments such as hot springs.

In a short paper, Gill [1] published a rigorous mathematical proof that buoyant flow in a vertical porous layer is linearly stable. He assumed that the boundaries of the porous layer were impermeable and kept at different constant temperatures T_1 and T_2. A recent study [2] reconsidered the analysis carried out by Gill on altering the boundary conditions of the system, namely by assuming open boundaries of the porous layer instead of impermeable walls. This change turned out to mean a significant difference with respect to the case analysed by Gill. In fact, the buoyant flow in the vertical porous slab displays a linear instability, which is absent if the boundaries are modelled as impermeable [2]. This analysis has been further developed by Barletta [3] by considering the case where both a pressure and a temperature difference is prescribed across the boundaries of the vertical porous slab.

Inspired by the pioneering paper by Gasser and Kazimi [4], several authors analysed the onset of convection in a horizontal porous layer under the influence of an internal heat source. Rhee et al. [5] collected experimental results on this convection system, by employing an inductively heated particulate bed. More recently, further results were obtained by He and Georgiadis [6], Khalili and Shivakumara [7], and by Nouri-Borujerdi et al. [8,9]. The effects of heterogeneity in the medium, have been investigated by Nield and Kuznetsov [10,11], by Kuznetsov and Nield [12–14], as well as by Shalbaf et al. [15]. Non-uniform internal heating has been analysed in a recent paper by Nield and Kuznetsov [16].

The aim of this paper is to further develop the analysis of instability in a vertical porous layer performed by Barletta [2] on considering a case where the uniform temperature gradient in the basic state becomes non-uniform due to the presence of an internal heat source. The basic stationary state is expressed by a pressure equal to the hydrostatic pressure, by a parallel velocity field directed along the vertical axis, and by a purely horizontal temperature gradient. The velocity and temperature profiles are given by second-degree polynomials in the horizontal coordinate. The linear stability analysis of the basic state is carried out versus general three-dimensional modes of perturbation. Neutral stability curves are drawn for different values of the heat source parameter. A shooting method is developed to compute the neutral stability data and to evaluate numerically the critical Rayleigh number and wave number. The effect of the internal heat source is expected to influence significantly not only the basic stationary solution, but also

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.04.063
0017-9310/

Nomenclature

a	dimensionless complex parameter, $\eta - i\omega$, Eq. (9)
$\hat{\mathbf{e}}_y$	unit vector in the y direction
$f(x), h(x)$	perturbation amplitudes, Eq. (9)
g	modulus of $\mathbf{g}$
$\mathbf{g}$	gravitational acceleration
$\hat{h}(x)$	modified perturbation amplitude, Eq. (13)
$\Im, \Re$	imaginary part, real part
k	wave number, $(k_y^2 + k_z^2)^{1/2}$
$\mathbf{k}$	wave vector, $(0, k_y, k_z)$
k_y, k_z	y and z components of the wave vector, Eq. (9)
K	permeability
L	channel width
N	number of steps used in the Runge-Kutta solver
p	pressure, Eq. (1)
p_0	reference pressure
$\dot{q}$	power per unit volume generated within the porous medium
Q	dimensionless parameter, Eq. (3)
$\hat{Q}$	dimensionless parameter, Eq. (13)
R	Rayleigh number, Eq. (3)
S	modified Rayleigh number, Eq. (11)
t	time, Eq. (1)
T	temperature, Eq. (1)
T_0	reference temperature, $(T_1 + T_2)/2$
T_1, T_2	boundary temperatures
$\mathbf{u}$	velocity, (u, v, w), Eq. (1)
(x, y, z)	Cartesian coordinates, Eq. (1)

Greek symbols

α	average thermal diffusivity
β	thermal expansion coefficient
δx	step-size used in the Runge-Kutta solver, $1/N$
ε	perturbation parameter, Eq. (6)
η	growth rate of the perturbations, $\Re(a)$
λ	average thermal conductivity
μ	dynamic viscosity
ν	kinematic viscosity
ξ_1, ξ_2	dimensionless parameters, Eq. (16)
σ	heat capacity ratio
ω	angular frequency of the perturbations, $-\Im(a)$

Subscripts, superscripts

$\sim$	perturbation fields, Eq. (6)
$'$	derivative with respect to x
b	basic solution
c	critical value

the stability analysis and the critical values for the onset of instability.

2. Governing equations

The Oberbeck-Boussinesq approximation is employed and the validity of Darcy's law for flow in a saturated porous medium is assumed [17]. Let us consider a plane and vertical porous slab with width L (see Fig. 1). We denote with $\mathbf{g}$ the gravitational acceleration and with g its modulus. From Fig. 1, we have $\mathbf{g} = -g\,\hat{\mathbf{e}}_y$, where $\hat{\mathbf{e}}_y$ is the unit vector along the y axis. The isothermal boundaries, $x = -L/2$ and $x = L/2$, are kept at different temperatures, T_1 and T_2, respectively. The medium is considered as homogeneous and isotropic, with local thermal equilibrium between the fluid and the solid phases. A dimensionless formulation can be introduced by means of a suitable scaling of the dimensional quantities,

$$\frac{1}{L}\,(x, y, z) \to (x, y, z), \qquad \frac{\alpha}{\sigma L^2}\,t \to t, \qquad \frac{K}{\mu\alpha}\,(p - p_0) \to p,$$
$$\frac{L}{\alpha}\,\mathbf{u} = \frac{L}{\alpha}\,(u, v, w) \to (u, v, w) = \mathbf{u}, \qquad \frac{T - T_0}{T_2 - T_1} \to T. \tag{1}$$

Here, $\mathbf{u}, p$ and T are the velocity, pressure and temperature, while (x, y, z) are the Cartesian coordinates. The difference $p - p_0$ is the local increment above the hydrostatic pressure p_0. Moreover, α is the average thermal diffusivity of the saturated porous medium, μ is the dynamic viscosity, K is the permeability, and σ is the heat capacity ratio of the saturated porous medium. The latter quantity is defined as the ratio between the average volumetric heat capacity of the saturated medium and the volumetric heat capacity of the fluid. In Eq. (1), the reference temperature T_0 is chosen as the mean value between T_1 and T_2, namely $T_0 = (T_1 + T_2)/2$.

Thus, the local mass, momentum and energy balance yield the dimensionless governing equations,

$$\nabla \cdot \mathbf{u} = 0, \tag{2a}$$

$$\mathbf{u} = -\nabla p + R\,T\,\hat{\mathbf{e}}_y, \tag{2b}$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \nabla^2 T + Q, \tag{2c}$$

where R is the Rayleigh number proportional to $T_2 - T_1$, and Q is a dimensionless parameter proportional to the intensity of the volumetric heat source, $\dot{q}$,

$$R = \frac{g\beta(T_2 - T_1)KL}{\nu\alpha}, \qquad Q = \frac{\dot{q}L^2}{\lambda(T_2 - T_1)}. \tag{3}$$

Since $\dot{q} > 0$, it is not restrictive to assume that both R and Q are both non-negative. In Eq. (3), ν denotes the kinematic viscosity of the

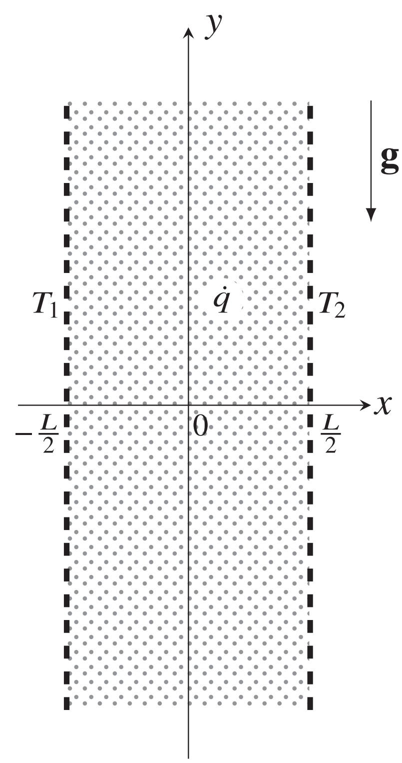

Fig. 1. Two-dimensional sketch of the vertical porous layer.