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Effect of gas properties on accelerated laminar boundary layer over a heated wall

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ABSTRACT

The paper reports on numerical investigation of an accelerated laminar boundary layer over a heated wall at Prandtl numbers from 0.23 to 2.39. Properties of three gases were simulated: air, helium-xenon mixture and superheated water vapor. The finite difference method was applied to solve a system of laminar boundary layer equations with variable gas properties. It has been shown that the influence of Prandtl number, viscosity, heat conductivity and heat capacity on the flow at studied conditions is secondary relative to gas density, which is the determinant factor for the overshoot magnitude and other flow parameters. Nevertheless, at insignificant differences in density distributions the overall decrease of Prandtl number and decrease of viscosity and heat capacity and increase of heat conductivity separately lead to an increase of maximal velocity within the boundary layer.

1. Introduction

Gases or liquids with different Prandtl numbers are applied in devices of gas-dynamic temperature stratification [1–6], heat exchangers [7] and for some other processes of heat transfer intensification. At the same time, there are conditions of accelerated flow over a heated surface, which produce an overshoot phenomenon, when due to the density difference over the boundary layer thickness the velocity within the layer is higher than the one of the main flow. A number of authors [8–10] observed for the first time the phenomenon in flows with chemical reactions. Paper [11] contains a short review of flows, where overshoot arises in the above and other conditions. The same work presents the results of studying of the boundary layer with favorable pressure gradient at heated surface streamlining by air. It is shown that the velocity maximum increases skin-friction coefficient and practically has no influence on Stanton number. In addition, it is proved that the reduced density near the wall is the main reason for the arising of overshoot. Nevertheless, there are unclear questions on the effect of Prandtl number, heat capacity, heat conductivity and viscosity on the overshoot magnitude and, as a consequence, on heat transfer and skin-friction in the boundary layer.

A number of studies [12–14] report on the influence of Prandtl number on velocity profiles in mixed convection flows. Their authors demonstrate that the overshoot is weak at $Pr = 7.0$. However, it is worth noting that all these researchers used Boussinesq approximation.

Bhattacharyya et al. showed that the decrease of Prandtl number from 1.0 to 0.2 increases the maximal streamwise velocity within the boundary layer. In accordance with the presented results the overshoot phenomenon is completely missing already at $Pr = 1.0$.

As it was mentioned above, the presence of velocity maximum within the boundary layer leads to an increase of skin-friction coefficient. Mureithi and Mason [16] demonstrated that in flows with mixed convection the increase of Prandtl number from 0.1 to 7.0 decreases skin-friction coefficient. At that, there is a decrease of heat transfer characterized by $Nu_x Re_x^{-0.5}$. Note that authors of [16] gave concrete parameters of overshoot occurrence. In addition, they proposed an alternative term for this phenomenon, calling it "super-velocity". Perhaps this word more closely reflects the nature of the phenomenon.

Papers [17–21], in contrast to the previous one, showed that the increase of Prandtl number decreases Nusselt number. At the same time, if one represents results of these investigations in the form $St = Nu_x/(Re_x Pr)$ [22] then Stanton number decreases with an increase of Prandtl number.

The paper presents numerical modeling of the accelerated laminar boundary layer over the heated wall in the presence of favorable pressure gradient at Prandtl number from 0.23 to 2.39. At that,

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Nomenclature

c_f	skin-friction coefficient
c_p	gas specific heat at constant pressure [J/(kg deg)]
$K = (\mu_e/\rho_e U_e^2) dU_e/dx$	stream acceleration parameter
L	length of wall [m]
P	pressure [Pa]
Pr	Prandtl number
q_w	heat flux at the wall [W/m^2]
Re_x	Reynolds number, based on streamwise coordinate x
Re_{int}	integral Reynolds number
Re_{x0}	Reynolds number, based on fixed initial velocity U_0
$St = q_w/\rho_e U_e c_p \Delta T$	Stanton number
T	temperature [K]
U, V	velocity components in the x, y directions respectively [m/s]
X	mass fraction of a component in the helium-xenon mixture
x, y	streamwise and normal coordinates relative to surface of streamlined body [m]

Greek symbols

α	the convergence angle of top plane of channel [degrees];
δ	thickness of hydrodynamic boundary layer [m], $U/U_e = 0.995$ or 1.005
δ_T	thickness of thermal boundary layer [m], $\Theta = 0.995$
λ	heat conductivity [W/(m · K)]
μ	dynamic viscosity [Pa · s]
$\Theta = (T - T_w)/(T_e - T_w)$	dimensionless temperature
ρ	density [kg/m^3]
$\psi = T_w/T_e$	temperature ratio

Subscripts

0	flow quantities at the start of flow
e	flow quantities in external flow
max	flow quantities at the point of maximum velocity
w	parameter at the wall

properties of three real gases were used: air, helium-xenon mixture and super heated water vapor. The goal of the investigation is to show the effect of working gas properties on velocity and temperature profiles, skin-friction and heat transfer distributions in the non-isothermal laminar boundary layer with favorable pressure gradient. There are many studies mentioned above used some assumptions for gas properties to simplify calculations and to generalize results. So, in our investigation we applied natural gas properties as much as possible. The analysis provided here covers only for the case of constant wall temperature. Study [23] showed that derivative flow characteristics for the case of $q_w = const$ in a certain cross-section coincide with the ones for the case of the flow with $T_w = const$ at the same temperature difference. Here, we accepted $K = 3 \times 10^{-6}$ since the effect of acceleration on the boundary layer was described in papers [11,24,25].

2. Flow configuration and modeling approach

We consider air flow in a plane convergent channel where the acceleration parameter $K = (\mu_e/\rho_e U_e^2) dU_e/dx$ remains constant over the entire channel length (Fig. 1). The lower wall is heated at a constant temperature T_w, higher than the main stream temperature T_e. The sloped top wall of the channel was assumed to be located sufficiently far from the lower channel wall, i.e. $h_0 \gg \delta$, so that the analysis can be focused on the accelerated hydrodynamic and thermal boundary layers, developing over the bottom surface. All parameters across the boundary layer and dimensionless characteristics are based on main flow quantities designated by subscript "e".

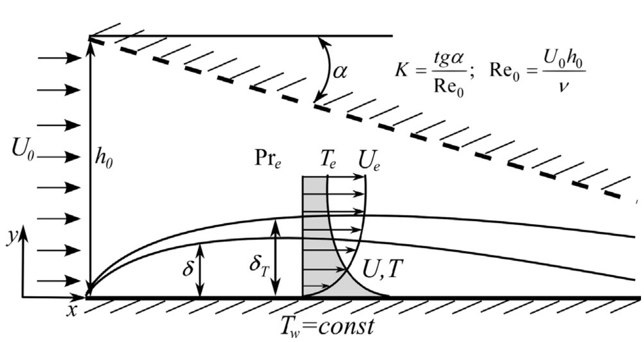

Fig. 1. Schematics of the flow considered.

2.1. Equations and boundary conditions

The flow considered can be well approximated by the parabolized two-dimensional momentum, continuity and energy equations for a steady laminar compressible boundary layer:

$$\rho U \frac{\partial U}{\partial x} + \rho V \frac{\partial U}{\partial y} = -\frac{dP}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial U}{\partial y}\right), \tag{1}$$

$$\frac{\partial(\rho U)}{\partial x} + \frac{\partial(\rho V)}{\partial y} = 0., \tag{2}$$

$$c_p \rho U \frac{\partial T}{\partial x} + c_p \rho V \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + U \frac{dP}{dx} + \mu \left(\frac{\partial U}{\partial y}\right)^2. \tag{3}$$

At the wall, the no-slip and constant temperature conditions were adopted:

$$y = 0: \quad U = 0, \quad V = 0, \quad T = T_w = const. \tag{4}$$

At the outer edge of the boundary layer, the velocity U_e was determined from the integration of the acceleration parameter $K = (\mu_e/\rho_e U_e^2) dU_e/dx$ accounting for variable viscosity and density of gas, with zero heat flux condition for the thermal field:

$$y \geqslant \delta: \quad U = U_e = -\left(K \int_0^x \rho_e/\mu_e dx - 1/U_0\right)^{-1}, \quad \partial T/\partial y = 0. \tag{5}$$

The initial velocity equaled $U_0 = 3$ m/s. Maximal Mach number and maximal Reynolds number were $M_{e\max} = 0.4$ and $Re_{x\max} = 10^7$ for all flow cases.

As it was mentioned above, we used three gases to vary properties in a wide range, namely, air [26], superheated water vapor [27–29] and helium-xenon mixture [30]. Changing the proportions in the helium-xenon mixture allowed varying its Prandtl number and appropriate thermodynamic properties. Pressure and temperature variations in ranges of P = 0.1–21 MPa and from $T_e = 653$ K to $T_w = 1073$ K determined properties of superheated water vapor. The ranges excluded areas with liquid water and supercritical liquid. Pressure equaled 0.1 MPa for air and helium-xenon flows.