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# Finite amplitude convection and heat transfer in inclined porous layer using a thermal non-equilibrium model

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Finite amplitude convection in a inclined porous layer heated from below is studied by using local thermal non-equilibrium (LTNE) as mathematical model which takes into account the heat transferred between the solid phase and the fluid phase. Consequently, in addition to Darcy-Rayleigh number $Ra$ and the inclination angle $\phi$, two further non dimensional numbers are introduced: the inter-phase heat transfer parameter $H$ and the porosity modified conductivity ratio $\gamma$. In a recent paper (Barletta and Rees, 2015), the linear stability analysis of the basic monocellular flow indicated that the inclination angle promotes the appearance of longitudinal rolls as the preferred mode of convection. The current paper focuses on the nonlinear evolution of longitudinal rolls in a supercritical regime of convection. A weakly nonlinear analysis, using a derived amplitude equation, is adopted to determine the nonlinear effects of the parameters $Ra, \phi, H$ and $\gamma$. The results indicate that in inclined layers (i) the nonlinearity decelerates the mean flow; (ii) the heat transfer, determined by the evaluation of the Nusselt number ($Nu$) at the layer boundary, corresponds to the one obtained for horizontal layers by scaling $Ra$ with $\cos\phi$, *i.e.* $Nu = Nu(Ra\cos\phi, H, \gamma)$; (iii) in accordance with existing laboratory experiments, the slope of $Nu$ is less than 2, where 2 is the value predicted by the local thermal equilibrium model, and the slope represents the derivative of $Nu$ with respect to the distance of the critical parameter from the threshold value for the onset of instability; (iv) increasing values of both $H$ and $\gamma$ produce an enhancement of the heat transfer across the layer. Finally, the comparison between the LTNE theoretical predictions and existing experiments conducted with various combinations of solid matrix and fluids suggests a possible alternative way to determine the heat transfer coefficient $H$.



## 1. Introduction

The problem of convective instability in a porous medium heated from below and saturated by a Newtonian fluid has been investigated extensively in the past. The work devoted to this area is well documented by the reviews of Nield and Bejan [1], Rees [2], Tyvand [3] and Barletta [4]. Among the experimental investigations aimed at visualizing the convective patterns and the temperature distributions we mention Elder [5], Combarnous [6], Close et al. [7], Shattuck et al. [8] and Howle et al. [9] for horizontal layers and Combarnous [10] when the porous layer is inclined to the horizontal.

For the horizontal configuration, the experimental results obtained with various combinations of solid particles and working fluids reveal that the heat transfer rate is not only a function of the Darcy-Rayleigh number $Ra$ but it can also be significantly affected by the structure of the medium and the fluid properties as well. Particularly, the measured slope of the Nusselt number versus the relative distance to the critical Darcy-Rayleigh number was found to depend on the solid/fluid combination and was less than the theoretical prediction of 2 derived by Joseph [11]. The explanation for the difference between theory and experiments may lie with the effects of finite heat transfer coefficient between fluid and solid phases. Combarnous and Bories [12] proposed a local thermal non-equilibrium (LTNE) model with two-energy equations, which introduces a finite inter-phase heat transfer coefficient and the ratio between fluid and solid conductivities as additional parameters. We mention that the growing volume of work with the LTNE model is well documented in [1,12–27].

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**Nomenclature**

| | |
|---|---|
| $a$ | dimensionless wave number |
| $A$ | amplitude of convection |
| $a_x; a_z$ | components of the dimensionless wave vector |
| $c$ | specific heat |
| $d$ | diameter of the beads |
| $\hat{\mathbf{e}}_x; \hat{\mathbf{e}}_y; \hat{\mathbf{e}}_z$ | unit vectors in the $(x, y, z)$-directions |
| $\mathbf{g}$; $g$ | gravitational acceleration vector; modulus of $\mathbf{g}$ |
| $h$ | volumetric inter–phase heat transfer coefficient |
| $H$ | volumetric dimensionless inter–phase heat transfer parameter, Eq. (2) |
| $J$ | heat flux |
| $k$ | thermal conductivity |
| $k_m$ | effective thermal conductivity, $\chi k_f + (1 - \chi)k_s$ |
| $K$ | permeability |
| $L$ | layer thickness |
| $\mathcal{L}, \mathcal{L}'$ | linear operators |
| LTE | local thermal equilibrium |
| LTNE | local thermal non-equilibrium |
| $N$ | nonlinear expression |
| $Nu$ | Nusselt number |
| $p$ | dimensionless pressure disturbance amplitude, Eq. (10) |
| $P$ | dimensionless pressure disturbances, Eq. (8) |
| $Ra$ | Darcy–Rayleigh number, Eq. (2) |
| $\mathfrak{Re}; \mathfrak{Im}$ | real part; imaginary part |
| $S; \tilde{S}$ | transformed Darcy-Rayleigh numbers, Eqs. (11) and (14) |
| $S_{LTE}$ | slope of Nusselt number according to local thermal equilibrium |
| $S_{LTNE}$ | slope of Nusselt number according to local thermal non-equilibrium |
| $t$ | dimensionless time, Eq. (2) |
| $T_0$ | temperature of the upper boundary |
| $T$ | dimensionless temperatures, Eq. (2) |
| $\mathbf{u}$ | dimensionless velocity vector, $(u, v, w)$, Eq. (2) |
| $\mathbf{U}$ | dimensionless velocity disturbance vector, $(U, V, W)$, Eq. (5) |
| $\tilde{V}$ | perturbations vector, $(\Theta_f, \Theta_s, P)^T$, Eq. (16) |
| $\mathbf{x}$ | dimensionless position vector, $(x, y, z)$, Eq. (2) |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | thermal diffusivity |
| $\alpha_m$ | effective thermal diffusivity, $k_m/(\rho c)_f$ |
| $\beta$ | thermal expansion coefficient |
| $\delta$ | coefficient, Eq. (36) |
| $\gamma$ | dimensionless parameter, Eq. (2) |
| $\Delta T$ | reference temperature difference |
| $\varepsilon$ | dimensionless perturbation parameter, Eq. (5) |
| $\theta$ | dimensionless temperature disturbance amplitudes, Eq. (10) |
| $\Theta$ | dimensionless temperature disturbances, Eq. (5) |
| $\lambda$ | dimensionless parameter, Eq. (2) |
| $\nu$ | kinematic viscosity |
| $\rho$ | density |
| $\varphi$ | arbitrary constant, Eq. (49) |
| $\phi$ | inclination angle to the horizontal |
| $\Phi$ | transformed angle, Eq. (11) |
| $\chi$ | porosity |
| $\omega$ | complex dimensionless parameter, Eq. (10) |
| $\tau$ | relaxation time, Eq. (41) |

*Superscript, subscripts*

| | |
|---|---|
| $\star$ | dimensional quantity |
| b | basic solution |
| c | critical value |
| f | fluid phase |
| s | solid phase |

On the other hand, in early experiments Bories and Combarnous [10] examined the secondary flow configurations of convection in a rectangular porous medium heated from below and inclined to the horizontal. The temperature recordings indicated that two main types of convective structures may be observed at the onset of convection. For small inclination angle $\phi$, the vortex patterns are polyhedral cells (i.e. oblique rolls). For higher values of $\phi$, the polyhedral cells are replaced by stationary longitudinal flow. The observed transition between the two types of convective patterns occurs at a critical angle $\phi_c \simeq 15^\circ$. Some hysteresis effects associated to this transition were also observed. However, the heat transfer measured through the boundaries for a large range of slopes ($0^\circ$–$60^\circ$) is found to be independent of the shape of the convective patterns, whether longitudinal rolls or polyhedral cells and a unique relation between Nusselt number and $Ra\cos\phi$ has been found. Caltagirone and Bories [28] examined the transition between the secondary flows in polyhedral cells and the longitudinal rolls both by three-dimensional numerical simulation of the problem and by performing a linear stability analysis of the basic state. A recent note by Nield [29] contains an interesting discussion on the results obtained by Caltagirone and Bories [28] and gives new insights into the question of the preferred patterns at the onset of the instability: rolls or polyhedral cells. Further results on the stability of an inclined porous layer were obtained by Storesletten and Tveitereid [30], Karimi-Fard et al. [31], Rees and Bassom [32], Rees et al. [33]. Karimi-Fard et al. [31] carried out an investigation of oscillatory instability for the case of double-diffusion. Rees and Bassom [32] defined a Squire-like transformation allowing a general study of normal modes with an arbitrary orientation. Storesletten and Tveitereid [30] included in the stability analysis the effect of anisotropy in the porous medium, while Rees et al. [33] extended this analysis by considering an arbitrary orientation of the principal axes of anisotropy. Recently Barletta and Rees [34] revisited the topic of instability in an inclined porous layer and performed a linear stability analysis in the framework of LTNE model by assuming an infinite extent of the porous cavity in the transverse and longitudinal directions. These authors showed that the longitudinal rolls are the preferred mode of instability at the onset of convection. The neutral stability for the longitudinal rolls is found to correspond to the one obtained for a horizontal layer, by scaling the Darcy-Rayleigh number with cosine of the inclination angle.

At the pore level, the interface between solid and fluid cannot display any discontinuity of temperature. However, when average temperatures are evaluated over a reference elementary volume (REV) for the solid and the fluid, these temperatures may well be different. This effect can arise in an unsteady regime, but also under steady conditions in cases where the thermal conductivities of the fluid and the solid are markedly different [1]. The LTNE model of heat transfer in porous media provides a description of how the different temperature fields of the solid phase and of the fluid phase interact.