International Journal of Heat and Mass Transfer 109 (2017) 746–754

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Predicting and analyzing interaction of the thermal cloaking performance through response surface method

Guoqiang Xu [a], Haochun Zhang [a,*], Quan Zou [a], Yan Jin [b]

[a] *School Energy Science and Engineering, Harbin Institute of Technology, Harbin 150001, China*
[b] *Institute of Thermo-Fluid Dynamics, Hamburg University of Technology, Hamburg D-21073, Germany*

ARTICLE INFO

Article history:
Received 14 December 2016
Received in revised form 7 February 2017
Accepted 19 February 2017

Keywords:
Thermal cloak
Entropy generation
Response surface method
Interaction effect

ABSTRACT

Thermal cloaks have been widely studied and experimented with. However, discussions on cloaking performance based on different design parameters effects are lacking. In this paper, we focus on predicting the cloaking performances of 2D thermal cloak schemes with different design parameters such as layer number, thickness, temperature difference, and relative thermal conductivity. The concept of response entropy, which is based on entropy generation analysis and is used to characterize the cloaking performance, is proposed. A regression model is established to predict the response entropy of the variational design parameters using response surface method (RSM). Five confirmation tests are conducted to verify the accuracy of the prediction. The relationship between the response entropy and the interaction effects of the design parameters are described with the help of 3D response surfaces using analysis of variance (ANOVA). In conclusion, we propose that a core-shell cloaking scheme with modest parametric values of 1–18 layers, a thickness of 1–5 mm, a small relative thermal conductivity, and large temperature difference can provide a better cloaking performance. This paper presents a theoretical prediction of the cloaking performance and provides suggestions on fabricating better thermal cloaks under different parameters.

1. Introduction

Invisibility cloaks, which are based on transformation optics and can hide an object without disturbing the surrounding optical [1,2], electromagnetic [3,4], acoustic [5,6], or elastodynamic [7,8] fields, have aroused considerable research interest in recent years. Many novel heat phenomena, which are similar to the cloaking structure in diffuse fields utilized by thermal cloaks, are being explored using both simulations and experiments. Spherical and prolate spheroid cloaks have been designed using transformation thermodynamics [9]. Since then, ideas of various thermal cloaking schemes have gained extensive attention. Two different ways of realizing thermal cloaking were proposed. The first one utilized transformation thermodynamics, i.e., it adopted coordinate transformation in different domains to map the thermodynamic properties onto a material distribution in order to create anisotropy in the cloaking region [10–13]. The second technique utilized the scattering-cancellation method [14–17] that is widely used in electromagnetic field applications [3,4]. In recent years, many peculiar thermal cloaking devices such as active pump [17], illusion [18], transparency [19], diodes [20], cloak-concentrator [21], camouflage [22], trap [23], and dissipation [24] were fabricated, based on the two methods mentioned above. Hu et al. [25] focused on the effect of heat convection with the ambient air and z-plane thickness to observe the 2D cloaking performance under specific conditions. Different kinds of cloaking schemes have been investigated extensively through simulations and experiments. In addition, various experimental conditions have been suggested for comparative studies, with a view to enhance heat conduction compared to convection [25]. Schittny et al. [26] summarized progress in steering diffuse light in turbid media, which was triggered by mathematical analogy between electrostatics, magnetostatics, stationary heat conduction, and stationary light diffusion, and presented an overview on core-shell invisibility cloaking. In all of the researches, the cloaking schemes always used different thicknesses and multi-material layers, which might have an influence on heat diffusion in the cloaking region. The variations in the relative thermal conductivities in the cloaking region and the temperature difference between the boundaries have an effect on the ability for heat diffusion and the dissipation distribution. Therefore, it is very important to probe and predict the cloaking performance under multiple variables for obtaining the expected performance from thermal cloaking devices.

---

[*] Corresponding author.
E-mail address: zhc5@vip.163.com (H. Zhang).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.02.056
0017-9310/© 2017 Elsevier Ltd. All rights reserved.

Nomenclature

A	areas of different parts (m^2)
a	area component of one ring in cloaking region
B	code of half of layer number
C	code of thickness (mm)
c	specific heat capacity ($J\,kg^{-1}\,K^{-1}$)
D	code of relative thermal conductivity ($W\,m^{-2}\,K^{-1}$)
E	code of temperature difference (K)
f	area fraction of copper in one ring (%)
k	number of variables
n	layer number
q	heat flux (W/m^2)
r	radius of measure points in the cloaking region (mm)
R	radius of cloaking region (mm)
$\dot{S}''_{gen}$	local entropy generation rate ($W\,m^{-2}\,K^{-1}$)
$\dot{S}_g$	entropy generation ($W\,K^{-1}$)
$\tilde{S}_g$	response entropy
T	temperature (K)
ΔT	temperature difference between hot and cold boundaries (K)
x	coded independent variables
y	response value

Greek symbols

α	regression coefficients
θ	azimuthal component (°)
κ	thermal conductivity ($W\,m^{-1}\,K^{-1}$)
ρ	density ($kg\,m^{-3}$)
ε	statistical error
δ	thickness of material layers (mm)

Subscripts

c	cloaking scheme
p	bare plate
I	part I
II	part II
III	part III

In this paper, we introduce entropy generation analysis [27], based on our previous work [24], to study the cloaking performance under multiple variables (thickness, number of layers of the multi-material, relative thermal conductivities in the cloaking region and temperature difference between the boundaries). The concept of response entropy is proposed to describe the cloaking performance in the cloaking region and the environmental suitability of the schemes. Furthermore, response surface methodology (RSM), which can provide an accurate prediction of the relationship between the system and the parameter interactions and is widely used in numerous manufacturing fields [28–30], was utilized to obtain the regression model of the cloaking performance (response entropy) with multiple variables through numerical simulations. Analysis of variance (ANOVA) was applied to confirm its effectiveness on statistics.

2. Theoretical method and models description

2.1. Theoretical method based on coordinate transformation

On the basis of the geometric transformation in Refs. [9–12,22,23,24], we consider the 2D heat transfer equation in the cloaking system without inner source as following [10,24]:

$$\rho' c' \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\kappa_r \frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\kappa_\theta \frac{\partial T}{\partial \theta}\right) \tag{1}$$

where ρ' is the density and c' is the specific heat capacity in the transformation domain, κ_r and κ_θ denote the components of thermal conductivities in radial and azimuth directions in the cloaking system, respectively.

The fundamental idea of transformation thermodynamics which was derived the thermal conductivities tensor κ in radius scope $R_1 < r < R_2$ is given in Refs. [10,12,24].

The radial component can be expressed as:

$$\kappa_r = \kappa_0 \left(\frac{R_2}{R_2 - R_1}\right)^2 \left(\frac{r - R_1}{r}\right)^2 \leqslant \kappa_0 \tag{2}$$

And the azimuthal component is:

$$\kappa_\theta = \kappa_0 \left(\frac{R_2}{R_2 - R_1}\right)^2 \geqslant \kappa_0 \tag{3}$$

where κ_0 is the thermal conductivity around the cloak ($W\,m^{-1}\,K^{-1}$).

The heat flux density distribution can be expressed by Fourier's law, so that heat flow in the transformation cloaking system corresponding Eqs. (2) and (3) is written as:

$$\begin{aligned} q_r &= -\kappa_0 \left(\frac{R_2}{R_2 - R_1}\right)^2 \left(\frac{r - R_1}{r}\right)^2 \frac{\partial T}{\partial r} \\ q_\theta &= -\kappa_0 \left(\frac{R_2}{R_2 - R_1}\right)^2 \frac{1}{r}\frac{\partial T}{\partial \theta} \end{aligned} \tag{4}$$

Considering the thermodynamic properties of the cloaking structure and its surrounding, we introduce entropy generation analysis [24,27] to provide the description of cloaking characteristic. For 2D transformation domain, the area entropy generation rate is:

$$\dot{S}''_{gen} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{q_r}{T}\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{q_\theta}{T}\right) \tag{5}$$

where $\dot{S}''_{gen}$ is the area entropy generation rate ($W\,m^{-2}\,K^{-1}$).

Substituting Eqs. (1) and (4) in Eq. (5), the area entropy generation rate of the cloaking system is presented as following:

$$\begin{aligned} \dot{S}''_{gen} = {} & \frac{\kappa_0}{T^2}\left(\frac{R_2}{R_2 - R_1}\right)^2 \left(\frac{r - R_1}{r}\right)^2 \left(\frac{\partial T}{\partial r}\right)^2 \\ & + \frac{\kappa_0}{r^2 T^2}\left(\frac{R_2}{R_2 - R_1}\right)^2 \left(\frac{\partial T}{\partial \theta}\right)^2 \end{aligned} \tag{6}$$

The entropy generation rate ($W\,K^{-1}$) of the cloaking system, which can be used to evaluate the randomness of the energy, varies with changes in the temperature gradient or the physical parameters at different positions. By considering the transformation local entropy generation rate distribution described above (Eq. (6)), the total entropy generation rate of the model is obtained by area integral (due to the 2D models). A denotes the corresponding areas of the local entropy generation rate:

$$\dot{S}_{gen} = \iint \dot{S}''_{gen} dA \tag{7}$$

The total entropy generation rate is comprised of three parts as shown in Fig. 1(a), i.e., the entropy generation rates of the background region (Part I), cloaking region (Part II), and the core region (Part III). Thus, Eq. (7) becomes