International Journal of Thermal Sciences 122 (2017) 281–291

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Thermal dispersion effects on heat transfer of laminar gas flow in a microtube filled with porous medium

Hossein Shokouhmand*, Pooya Bagherzade, Mohammad Fazli, Ali Shomali

School of Mechanical Engineering, College of Engineering, University of Tehran, P. O. Box: 11155-4563, Tehran, Iran

A R T I C L E   I N F O

Article history:
Received 1 February 2017
Received in revised form
28 July 2017
Accepted 1 September 2017

Keywords:
Microtube
Porous medium
Slip flow
Thermal dispersion
Local thermal non-equilibrium

A B S T R A C T

In this paper, dispersion effects on forced convection heat transfer in a gaseous slip flow through a microtube filled with anisotropic porous medium are investigated semi-numerically. Microtube is subjected to constant heat flux and a local thermal non-equilibrium condition is assumed. Rarefaction effects are taken into account by applying first-order velocity slip and temperature jump boundary conditions. Dispersion is proven to have a significant impact on heat transfer in rarefied gases. Temperature distributions for both solid and fluid phases are obtained. Nusselt number variation with respect to porous shape factor and Biot number is illustrated, indicating enhancement up to 10% in predicted heat transfer due to dispersion. The influence of Reynolds number on thermal dispersion is proven to be insignificant. Dispersion plays a significant role as gas becomes more rarefied. It is seen that an increase in shape factor of a porous media can improve *Nu* for a non-rarefied gas, but as rarefaction effects grow, this can affect heat transfer negatively. Also, effects of dispersion on heat transfer diminishes for higher values of Biot Number due to the overcoming of interstitial heat transfer to thermal dispersion effect.

1. Introduction

Heat transfer in microscale, such as heat transfer by gaseous flows through porous microchannels, has become a research interest in recent years due to its growing application and capabilities in cooling of Nano and micro-electronic devices. Microchannel flow has been the subject of numerous theoretical, numerical, and experimental studies because of its inherent complexity caused by rarefaction effect, surface properties, intermolecular collisions, and other microscale phenomena.

The degree of rarefaction, evaluated by Knudsen number (Kn), has a significant effect on modeling and analyzing physics of the microchannel problem. An increase in Knudsen number leads to inaccuracy of continuum assumption, therefore instead of continuum ($Kn < 0.01$), other regimes such as slip flow ($0.01 < Kn < 0.1$), transient ($0.1 < Kn < 10$), and free molecular regime ($Kn > 10$) should be considered respectively [1,2]. Thus the physical model conducted to analyze the problem differs from Navier-Stokes. In this case, Navier-Stokes with modified wall boundary conditions for slip flow regime and Lattice-Boltzmann or molecular dynamics (MD) methods (for a more rarefied gas) must be applied [3].

Several types of analytical and experimental velocity and temperature jump boundary conditions have been derived for slip flow regime, including first order, second order, and hybrid slip model [2]. Maxwell [4] extracted first order slip model and obtained slip coefficients by defining tangential momentum accommodation coefficients which refer to surface molecular absorption/reflection. Acquiring accommodation coefficients with different approaches and in different Knudsen numbers has been the subject of numerous studies [5–7].

Even more theoretical challenges are associated with Porous microchannels due to interstitial heat transfer between porous medium matrix and gas flow. Local thermal equilibrium (LTE) and local thermal non-equilibrium (LTNE) are known as the most fundamental models that can offer a general explanation of microchannel problems. LTE model assumes no temperature difference between solid and gas phases, while LTNE considers two different temperature distributions for each phase. Shokouhmand et al. [8] conducted a numerical simulation of LTE model on Darcy-Brinkman-Forchheimer flow in micro/nano-tubes filled with porous medium. Several studies have focused on the validity of LTE and LTNE plus the physical and geometrical parameters that affect these two models [9–11]. Generally, LTNE is used when heat transfer between solid and gas phases is low and the temperature

---

* Corresponding author.
E-mail address: hshokoh@ut.ac.ir (H. Shokouhmand).

http://dx.doi.org/10.1016/j.ijthermalsci.2017.09.002
1290-0729/

difference between them is not negligible [12]. Nield and Kuznetsov [13] used LTE model and extensively discussed thermally developing forced convection of a rarefied gas through a parallel and circular microchannel with constant wall heat flux. Hashemi et al. [14] applied LTE model and tackled the issue of non-Darcian flow through an annular microchannel filled with porous medium. They derived Nusselt correlation and demonstrated the variation of Nusselt number with aspect ratio, porous shape parameter, and Knudsen number, resulting in a decrease in Nusselt number by Knudsen value increment. Recently, various studies have applied LTNE model microfluidics and compared the results with LTE. For instance, Haddad et al. [15] investigated forced convection of a rarefied gas through a parallel and circular microchannel with constant wall heat flux. Wang et al. [12] obtained the analytical solution for a gaseous flow through a porous microtube subjected to a constant wall heat flux under LTNE condition. They demonstrated variations of friction coefficient and Nusselt number with respect to Biot number, porous shape factor, and Knudsen number. In the same vein, Wang et al. [16] discussed the same parameters for LTNE model applied for an annular microchannel. Many of these analytical solutions use basic assumptions for simplifying momentum and energy differential equations, which must be validated through experimental studies, numerical simulations, and more accurate theoretical studies on the subject. Some of these assumptions neglect viscous dissipation, axial conduction, thermal dispersion, radiative heat transfer, natural convection, thermal creep, and wall roughness effects [12]. Bigham et al. [17] numerically investigated the combined effects of rarefaction, creeping flow, and viscous dissipation on hydrodynamically and thermally flow characteristics of a microchannel. Shokouhmand et al. [18] studied friction factor and aspect ratio effects on convective heat transfer and observed an increase in heat transfer coefficient by increasing the relative roughness of channel. Consequently, microscale heat transfer of gaseous flows in microchannels demands more studies in order to provide a general and more thorough insight into this phenomenon.

Thermal dispersion includes several micro and macro trends such as randomness of porous matrix geometry, channeling effect, recirculation of fluid, and etc. [19] It is explained as a mixture of fluid interstitial velocity in pore scale and molecular diffusion which is caused by the complexity of random tortuous paths provided for a fluid stream in porous matrix [20]. Thermal Dispersion has been considered in many studies and is found to affect the accuracy of predicted results significantly [21–23]. Bear [24] and Scheidegger [25] generalized the dispersion tensor from a theoretical point of view. Subsequently, Poreh [26] derived the tensor form of dispersion for Isotropic and Axisymmetric Mediums and discussed its dependency to pore size, Reynolds, and Peclet numbers for particular cases of high and low Reynolds numbers. More examples on this subject are Nakayama's [27] and Koch's [28] investigations. According to Hsu et al. [29], thermal dispersion effect can be modeled as an additive thermal diffusion in volume-averaged energy equation which differs for low and high Reynolds numbers. The model describes this additive term depending on the first and second order of the Peclet number for high and low Reynolds number respectively. This model has been utilized in a study by Jiang and Ren [23] on a porous channel. Amiri and Vafai [22] and Alazmi and Vafai [21] employed empirical correlation of effective thermal conductivity, developed by Wakao and Kaguei [30], which assumes a linear proportionality between dispersion thermal diffusivity and Peclet number for high Reynolds numbers in both transverse and longitudinal directions. Kuwahara et al. [31] numerically obtained the transverse coefficient of thermal dispersion by analyzing a lattice of rods placed regularly in an infinite space with LTE assumption. They also studied the tortuosity conductivity variations by Peclet Number. Moreover, Kuwahara [32] determined the longitudinal dispersion coefficient by a similar method. Nakayama et al. [33] determined an analytical effective thermal conductivity correlation in the longitudinal direction for both laminar and turbulent flow regimes with LTNE assumption.

Although numerous researches have been conducted in order to determine the effects of thermal dispersion in heat transfer, none of them studied these effects on a micro-scale geometry in which rarefaction effects are significant. In this paper, the effects of thermal dispersion in forced convection of a rarefied gaseous fully developed flow through a porous microtube subjected to constant heat flux are investigated semi-numerically. Interstitial heat transfer within the porous matrix is assumed to be indispensable, therefore, LTNE assumption is implemented. Transverse thermal dispersion is modeled as an effective thermal conductivity by interpreting analytical correlation of Nakayama et al. [33]. A comparison between two distinct cases of considered and neglected thermal dispersion is presented. The effects of rarefaction and various physical parameters like shape factor, Biot number, and Reynolds number on heat transfer are studied as well.

2. Analysis

A schematic view of the problem is presented in Fig. 1 in which microtube of radius R filled with porous medium and constant wall heat flux q_w as boundary condition is illustrated. Following assumptions are invoked in this study:

- The flow is one-dimensional along the z-axis, laminar, steady, and incompressible; flow domain is also anisotropic.
- Brinkman-extended Darcy model is invoked.
- Local thermal non-equilibrium model is considered.
- Axial conduction, natural convection, and radiative heat transfer are negligible.
- Flow is fully developed both hydrodynamically and thermally.
- Thermal dispersion is taken into account by an effective thermal conductivity.
- The porous media is in an unconsolidated form made of convex beads.

Based on considered Knudsen numbers of this study, slip flow regime is implemented. Hence, velocity slip and temperature jump boundary conditions are invoked as follows [34].

$$u^*_{f,w} - u^*_w = -\alpha_v \lambda \left.\frac{\partial u^*}{\partial r^*}\right|_{r^*=R^*} \tag{1}$$

$$T^*_{f,w} - T^*_w = -\alpha_t \lambda \left.\frac{\partial T^*_f}{\partial r^*}\right|_{r^*=R^*} \tag{2}$$

where λ is molecular mean free path and u^* and T^*_f indicate velocity and temperature of the flow. Here, asterisks denote

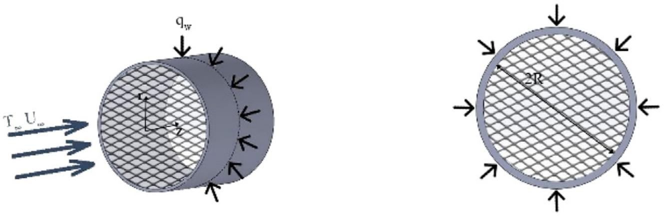

Fig. 1. A schematic of the problem.