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Revisit on natural convection from vertical isothermal plate arrays–effects of extra plume buoyancy☆

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In this study, natural convection from vertical isothermal parallel-plate arrays is examined using 2-D steady-state numerical analysis. Extended computation domains encompassing the single- or multi-channel plate arrays are adopted. Also investigated are the consequences of using computation domains without the inlet extension or/and the outlet extension. The plate height is fixed at 100 mm, the plate spacing is 7 mm, and the plate thickness is 1 mm. The Elenbaas Rayleigh number *Ra'* is fixed at 62.8. The results show that setting the inlet boundary at the entrance of the channel would ignore the separation formed near the outer entrance corner and the associated flow resistance; while setting the outlet boundary at the exit of the channel would omit the extra buoyancy provided by the hot plume above the arrays. In multi-channel arrays, the average heat transfer coefficients in different individual channels (*hs*) are different from each other. The *h* is the highest in the central channel and the lowest in the edge channel. Except for the edge channels, the *h* values are higher than that of a single channel. The differences in *h* between the central and the edge channels increase with increasing number of channels. This phenomenon can be ascribed to (1) the stronger extra hot-plume buoyancy in the inner plume region and (2) the higher entrance-separation resistance in the outer channels. The overall convection heat transfer coefficients in the plate arrays increase with increasing number of plate channels. The conventional assumption that all the channels of a multi-plate array have similar heat transfer performance needs re-evaluation.

1. Introduction

Natural convection from a vertical parallel plate array, one of the cornerstone problems of natural convection heat transfer, has been widely studied. This heat transfer configuration can be found in many practical applications, such as heat exchanger, heat sinks for electronics or LED cooling, etc. Elenbaas [1] pioneered the measurement of average convection heat transfer coefficient (h) on the inner isothermal walls of two isolated square parallel plates. It was assumed that the flow characteristics in all the channels of a parallel-plate array are similar and can be represented by a single channel between two parallel plates. Examined parameters included the plate spacing (b), the plate height ($H = 5.95$, 12, and 24 cm), the plate temperature (T_w), and the inclination angle. The 3-D experimental data for square plates were transformed with approximations into a 2-D (for infinitely long plates) empirical correlation between Nusselt number (Nu) and the Elenbaas Rayleigh number (Ra') over $0.2 < Ra' < 1 \times 10^5$:

$$\mathrm{Nu} = \frac{1}{24}\mathrm{Ra}'\left[1 - e^{-\frac{35}{Ra'}}\right]^{3/4} \tag{1}$$

where $Nu \equiv \frac{hb}{k}$, $Ra' \equiv \frac{b}{H} Ra = \frac{b}{H}\ \frac{b^3 g\beta(T_w - T_\infty)}{\nu\alpha}$.

An empirical formula for the optimum plate spacing was further proposed based on the experimental data. The optimum spacing to yield maximum total heat transfer rate of the plate array was located at $Ra' = 46$. Since then, almost all numerical or theoretical studies on the natural convection form a vertical multi-plate array were simplified to 2-D analyses between two parallel plates [2–15].

Bar-Cohen and Rohsenow [2] utilized the circumstances that the flows between two vertical parallel plates are bounded by two extreme situations: a fully-developed flow within two very close plates and a single-plate boundary layer flow with an infinite plate distance. An integrated formula can be obtained by combining the

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Nomenclature

A_w inner wall area (m^2)
b plate spacing (mm)
g gravitational acceleration, 9.81 (m/s^2) along negative y-direction
Gr Grashof number, $Gr = \frac{b^3 g\beta\rho^2(T_w - T_\infty)}{\mu^2}$
h average heat transfer coefficient for an individual channel, $h = Q/A_w(T_w - T_\infty)$ (W/m^2K)
h_n average heat transfer coefficient in the nth channel of a multi-channel plate array (W/m^2K)
h_o overall heat transfer coefficient for a multi-channel plate array (W/m^2K)
h_s average heat transfer coefficient for a two-plate single channel (W/m^2K)
H plate height (mm)
k thermal conductivity of air (W/mK)
L_b distance between plate's upper end and upper boundary of computational domain (mm)
L_s distance between side-end plate and side boundary of computational domain (mm)
L_t distance between plate's lower end and lower boundary of computational domain (mm)
$\dot{m}$ mass flow rate in an individual channel in multi-channel plate array (kg/s)
$\dot{m}_o$ overall mass flow rate in multi-channel plate array (kg/s)
$\dot{m}_s$ mass flow rate in two-plate single channel (kg/s)
n channel number, $n = 1$ for the middle channel
N total number of channels in a plate array
Nu Nusselt number defined as $Nu = \frac{hb}{k}$
p pressure (Pa)
q_w heat flux on the channel wall per unit depth (W/m^2)
Q heat transfer rate in a channel (W)
Ra Rayleigh number, $Ra = \frac{b^3 g\beta(T_w - T_\infty)}{\nu\alpha}$
Ra' Elenbaas Rayleigh number, $Ra' = \frac{b}{H}\ \frac{b^3 g\beta(T_w - T_\infty)}{\nu\alpha}$
t plate thickness (mm)
T temperature (K)
T_w inner plate wall temperature (K)
T_∞ ambient air temperature (K)
u, v velocities in x- and y-direction, respectively
x, y coordinate directions

Greek symbols
α thermal diffusivity (m^2/s)
β thermal expansion coefficient (1/K)
ν kinematic viscosity (m^2/s)
ρ air density (kg/m^3)
ρ_∞ air density under ambient temperature (kg/m^3)

two limiting asymptotic analytic solutions. This principle was applied for four different heating arrangements: symmetric isothermal, asymmetric isothermal, symmetric isoflux, and asymmetric isoflux. For the symmetric isothermal situation, the following formula was proposed:

$$Nu = \left(\frac{576}{Ra'^{\,2}} + \frac{2.873}{\sqrt{Ra'}}\right)^{-1/2}. \tag{2}$$

This relation matches well with the 2-D empirical *Nu* formula of Elenbaas [1]. With the assumption that all channels in a multi-channel plate array are similar, the formulae for the optimal plate spacing were derived for these four heating arrangements. Among them, the optimum spacing for symmetric isothermal plate arrays occurs at $Ra' = 54.3$, slightly higher than Elenbaas' value. These formulae have been widely used for the optimum design of vertical fin arrays [16,17].

Bodoia and Osterle [3] and Aung et al. [4] performed 2-D boundary-layer analysis between two isothermal parallel plates. The computation domain was set between the entrance and the exit of the channel. A uniform flow velocity was imposed at the channel entrance. Their 2-D numerical *Nu* results agreed fairly well with 3-D experiments of Elenbaas [1], with about 10% over-estimations for $Ra' > 400$. The discrepancies were ascribed to the assumption of uniform velocity at the channel entrance which ignored the entrance flow resistance [4]. In addition, Bodoia and Osterle [3] found their *Nu* calculations significantly lower than the 3-D data of Elenbaas [1] at low *Ra'* values. This discrepancy was attributed to the side leakage effects in the 3-D experiments when plate spacings were small. Anand et al. [5] also adopted the boundary-layer approximations and set the computation domain between the entrance and the exit of the channel. Wang and Pepper [6] formulated with the elliptic Navier-stokes equations and energy equation, but the computation domain was still set between the channel entrance and exit.

To account for the entrance flow resistance, an extended inlet subdomain as well as elliptic governing equations was employed in Refs. [7–10]. But in these works, the outlet boundary was set at the channel exit. Martin et al. [9] focused on the low *Ra'* regime for an isothermal parallel-plate channel, where streamwise diffusion and heat conduction to the upstream air and adjacent surfaces become influential. They found the low-*Ra'* heat transfer dependent on the particular inlet and outlet configuration used. Naylor et al. [10] reported entrance separation as *Ra'* increased to 291.7, owing to the edge effect with inlet flow turning. This separation was shown to have an adverse effect on the local heat transfer near the entrance.

Knowing that setting the outlet boundary at the channel exit excludes the effect of the hot outflow region, Chang and Lin [11] and Andreozzi et al. [12] adopted I-shaped computation domains with extended inflow and outflow subdomains in their transient 2-D simulation for the channel flow between two symmetrically-heated parallel plates. Morrone et al. [13] also used I-shaped computation domains in their analysis aiming to obtain an empirical correlation for the optimum plate spacing. Shyy et al. [14] adopted a convergent inflow subdomain and a divergent outflow subdomain in the 2-D steady analysis. The inflow boundary conditions utilized the inviscid flow generated by a line source on the centerline at the channel inlet, coupled with free entrainment boundaries along the side. Similarly, the outflow boundary conditions correspond to a line source plume coupled with free entrainment boundaries along the side. Ramanathan and Kumar [15] sidestepped the problems of open boundary conditions at the inlet and the exit by considering a large virtual enclosure encompassing the flow channel. However, the channel flow might not be completely free from the natural recirculation associated with the enclosure. It is noted that the above studies [1–15] only treated a single vertical channel.

To explore the thermal and flow characteristics in multi-channel plate arrays, Floryan and Novak [18] simulated isothermal plate arrays with two, three, or an infinite number of channels, adopting