Journal of Process Control 58 (2017) 139–151

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

Improved solutions for ill-conditioned problems involved in set-membership estimation for fault detection and isolation

Laleh Ravanbod [a], Carine Jauberthie [b,c,*], Nathalie Verdière [d], Louise Travé-Massuyès [b,e]

[a] Université du Havre, 25 rue Philippe Lebon, BP 540, 76058 Le Havre Cedex, France
[b] CNRS, LAAS, 7, avenue du Colonel Roche, F-31400 Toulouse, France
[c] Université de Toulouse, UPS, LAAS, F-31400 Toulouse, France
[d] Normandie Univ, UNIHAVRE, LMAH, FR-CNRS-3335, ISCN, 76600 Le Havre, France
[e] Université de Toulouse, LAAS, F-31400 Toulouse, France

ARTICLE INFO

Article history:
Received 14 February 2016
Received in revised form 22 July 2017
Accepted 7 August 2017

Keywords:
Uncertain dynamic systems
Bounded noise
Nonlinear systems
Conditioning problem
Fault detection and diagnosis

ABSTRACT

Set-membership (SM) estimation implies that the computed solution sets are guaranteed to contain all the feasible estimates consistent with the bounds specified in the model. Two issues often involved in the solution of SM estimation problems and their application to engineering case studies are considered in this paper. The first one is the estimation of derivatives from noisy signals, which in a bounded uncertainty framework means obtaining an enclosure by lower and upper bounds. In this paper, we improve existing methods for enclosing derivatives using Higher-Order Sliding Modes (HOSM) differentiators combining filtering. Our approach turns the use of high order derivatives more efficiently especially when the signal to differentiate has slow dynamics. The second issue of interest is solving linear interval equation systems, which is often an ill-conditioned problem. This problem is reformulated as a Constraint Satisfaction Problem and solved by the combination of the constraint propagation Forward Backward algorithm and the SIVIA algorithm. The two proposed methods are tested on illustrative examples. The two methods are then used in a fault detection and isolation algorithm based on SM parameter estimation that is applied to detect abnormal parameter values in a biological case study.

1. Introduction

This paper focuses on two mathematical problems encountered in several engineering problems: the calculus of uncertain derivatives and conditioning problems involved in solving linear interval equation systems. These two issues are addressed in a set-membership (SM) framework in which uncertainties are characterized by simple bounds. SM estimation methods advantageously provide guaranteed solutions, meaning that the computed set estimates are guaranteed to contain all the feasible estimates consistent with the specified bounds. SM estimation methods have been successfully applied to many tasks [1–4]. SM estimation can be based on interval analysis that was introduced by [5] and several algorithms have been proposed along this line for nonlinear systems (for more details, see [6,4,7]). The approaches dedicated to linear systems are rather based on ellipsoid shaped methods (for example [8]), parallelotope or zonotope based methods [9]. The advantage of providing guaranteed results is unfortunately often stained by the overestimation of the results. In this respect, it is mandatory to carefully analyse every single step of SM estimation methods to compensate for possible spurious uncertainty propagation. In particular, the resolution of estimation problems often requires to evaluate successive derivatives of signals, which is known to be a tedious numerical problem, and/or to solve systems of linear interval equations, which must often handle ill-conditionment. This paper deeply inspects these two problems and proposes improved solution methods.

Derivative estimation from noisy signals given by discrete measurement samples is an important and difficult task in numerical analysis, signal processing and control. It is well-known that it is an ill-posed problem. In the literature, several classes of derivative estimation methods have been proposed. The first class consists in approximating the signal by polynomials using least-square estimation and adding a regularization criterion [10,11]. Another class consists in approximating the signal by a truncated Taylor expansion and to operate in either the Mikusinski field [12,13] or the distribution space [14]. Yet another class is based on sliding-mode

* Corresponding author at: CNRS, LAAS, 7, avenue du Colonel Roche, F-31400 Toulouse, France.
E-mail address: cjaubert@laas.fr (C. Jauberthie).

http://dx.doi.org/10.1016/j.jprocont.2017.08.003
0959-1524/

differentiators [15,16]. In the frame of interval analysis, apart from the classical finite difference that has been extended [17], there are few works guaranteeing enclosures of successive derivatives. Nevertheless, the estimation of derivatives is essential in many basic algorithms such as the evaluation of centered inclusion functions, Newton contractors, etc. In [18], the Higher-Order Sliding Modes (HOSM) differentiators developed by Levant in 1998 have been used to obtain an exact enclosure of derivatives required by a fault detection method. The drawback of these differentiators is that only the first derivative is calculated with reasonable overestimation contrary to higher order derivatives. In this paper, we propose to combine the methods developed by Levant with a zero-phase low-pass filtering algorithm, guaranteeing a robust enclosure of the successive derivatives even for high orders. Some examples are given and confirm the robustness of the method.

The problem of solving a linear interval system is considered in the second part of the paper. Although the system is linear, this problem is NP-hard due to the presence of interval matrices [19]. Some algorithms for solving interval linear systems return a box containing the convex hull of the solutions, which is not the minimal enclosure [20]. Unlike direct algorithms for enclosing the solution of an interval linear system [21,22], we rely on an iterative method [20] because it advantageously allows one to control the computation time. Our method assumes that an initial enclosure is known from the knowledge of the system and applies contractions using the Forward Backward Propagation algorithm [23] based on interval Gauss-Seidel iterations. Then we use this solution to initialize a branch and bound algorithm, based on Set Inversion Via Interval Analysis (SIVIA), which further improves the result. Some details on contractors and the SIVIA algorithm can be found in [23,6,24].

The last part of the paper integrates the two proposed methods to improve an off-line previously developed FDI procedure for nonlinear dynamical systems based on SM parameter estimation [25]. The developed FDI procedure requires the estimation of derivatives, sometimes of high order, from discrete measurement samples and the solution of linear systems of interval equations involving blocks of parameters.

The paper is organized as follows. Section 2 briefly introduces the problem of enclosing successive derivatives of a signal corrupted by bounded noise and the new method that we propose based on HOSM differentiation and filtering is presented. Bounded noises are a natural way to model the realistic stochastic fluctuations of a biological system, for example, that are caused by its interaction with the external world. Bounded noise is also well-adapted to sensors tolerances. The proposed method is applied to classical examples which highlight its advantages. Section 3 explains the problem of solving linear interval systems, and exhibits the sources of ill-conditioning. Our method based on contractors and set inversion is presented. Through some examples, the results obtained by the proposed scheme are compared with those obtained by classical solvers. In Section 4, the previously developed methods are applied in a SM algorithm for FDI in nonlinear dynamical systems. The application of this algorithm to a cell exchange model is reported. Finally, Section 5 concludes the paper and provides some perspectives.

2. Derivative estimation

The aim of this section is to present a differentiator that provides robust exact intervals containing the successive derivatives of a signal corrupted by a bounded noise whose bounds are supposed known. Bounded noise is a convenient way to characterize uncertainty when a more informative statistical model is not known. It accounts for nonlinear phenomena like saturation which are often encountered in practice. For example, bounded noise is a natural way to model the realistic stochastic fluctuations of a biological system, for example, that are caused by its interaction with the external world [26].

The differentiator has been proposed by Levant in 1998 [15] under the name of the Higher-Order Sliding Modes (HOSM) differentiator. It is presented in the first part of this section. Although other differentiators like the asymptotic differentiator [27] or the high gain observer [28] have been proposed, the differentiator of [15] advantageously provides exact differentiation in finite-time of noise-free signals satisfying some Lipschitz constraint. That is why we have built on this work.

Despite its advantages, the Higher-Order Sliding Modes (HOSM) differentiator has the drawback that for a high derivative order or for a significant noise, the intervals containing the derivative estimates are very overestimated. Thus we propose an original approach that provides tighter interval enclosures of derivatives thanks to a low pass filter. This smoothing makes the enclosure of high order derivatives more efficient specially when the signal to be differentiated has slow dynamics.

Let us consider the following standard notations and definitions [6]. A real interval $[x] = [\underline{x}, \quad \bar{x}]$ is a connected and closed subset of $\mathbb{R}$. The notation $\mathbf{x}$ defines the real vector $\mathbf{x} = (x_1, \ldots, x_n)^T$, where ^{T} stands for the transpose of the considered vector whereas $[\mathbf{x}]$ defines an interval vector, also called a box. w represents the interval width. If $[x] = [\underline{x}, \quad \bar{x}]$ then $w([x]) = \bar{x} - \underline{x}$. In the same manner $w([\mathbf{x}]) = \max \quad (\bar{\mathbf{x}} - \underline{\mathbf{x}})$.

2.1. HOSM differentiator

In the works [15,29,16] concerning HOSM differentiators, the signal $y(t)$ to be differentiated is considered as a function defined on $[0, +\infty[$. It is supposed to be composed of a bounded Lebesgue-measurable noise $e(t)$ (bounded by a positive constant α) with unknown features and an unknown base signal $y_0(t)$ with the mth derivative having a known Lipschitz constant $C > 0$. The m successive derivatives of the signal $y(t)$, i.e. $y^{(1)}(t), \ldots, y^{(m)}(t)$ are estimated by $z_1(t), \ldots, z_m(t)$ for $t \geq t_c$ as described below:

$$\begin{cases} \dot{z_0} = & v_0, \\ v_0 = & -\lambda_0 |z_0 - y|^{\frac{m}{(m+1)}} \operatorname{sign}(z_0 - y) + z_1, \\ \dot{z_1} = & v_1, \\ v_1 = & -\lambda_1 |z_1 - v_0|^{\frac{(m-1)}{m}} \operatorname{sign}(z_1 - v_0) + z_2, \\ & \vdots \\ \dot{z}_{m-1} = & v_{m-1}, \\ v_{m-1} = & -\lambda_{m-1} \left| z_{m-1} - v_{m-2} \right|^{\frac{1}{2}} \operatorname{sign}(z_{m-1} - v_{m-2}) \\ & + z_m, \\ \dot{z}_m = & -\lambda_m \operatorname{sign}(z_m - v_{m-1}), \end{cases} \tag{1}$$

where $\lambda_j \in \mathbb{R}, \quad j = 0, \ldots, m$, represent the differentiator parameters and t_c is the convergence time of the differentiator which depends on λ_j. Generally, the parameters $\lambda_j, j = 0, \ldots, m$, are chosen experimentally (for more details, see [15,29]). In other words, these parameters must be re-evaluated for every new signal. This is not surprising because it is well known that an ideal differentiator, i.e. that can differentiate any signal, does not exist.