Journal of Process Control 57 (2017) 127–139

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

Control-relevant nonlinearity measure and integrated multi-model control

Jingjing Du[a,*], Tor Arne Johansen[b]

[a] College of Internet of Things Engineering, Hohai University, Changzhou, 213022, China
[b] Department of Engineering Cybernetics, Norwegian University of Science and Technology, NO 7491, Trondheim, Norway

ARTICLE INFO

Article history:
Received 5 April 2017
Received in revised form 11 July 2017
Accepted 11 July 2017

Keywords:
Control-relevant nonlinearity measure
Gap metric
Weighting method
Integrated multi-model control
CSTR

ABSTRACT

A control-relevant nonlinearity measure (CRNM) method is proposed based on the gap metric and the gap metric stability margin to measure the nonlinear degree of a system once a linear control strategy is selected. Supported by the CRNM method, an integrated multi-model control framework is developed, in which the multi-model decomposition and local controller design are closely integrated, model redundancy is avoided, computational load is reduced, and dependency on a prior knowledge is reduced. Besides, a $1/\delta$ gap-based weighting method is put forward to combine the local controllers. On one hand, the $1/\delta$ gap-based weighting method has merely one tuning parameter and can be computed off-line; on the other hand, it is sensitive to the tuning parameter, flexible and easy to tune. Two continuous stirred tank reactor (CSTR) systems are investigated. Closed-loop simulations validate the effectiveness and benefits of the proposed integrated multi-model control approach based on CRNM.

1. Introduction

Virtually all chemical processes are nonlinear. However, most of them are handled using linear analysis and design techniques because of operating around an equilibrium point, so that the development and implementation of a controller can be largely simplified [1]. Nevertheless, in some important cases, the linearity assumption does not hold and linear controllers are invalid. Then nonlinear controllers are necessary. Therefore, from the perspective of controller design, there is a need for nonlinearity measures, which quantify the nonlinearity extent of a process instead of merely judging a system as linear or nonlinear. Thus we can decide whether a linear controller is adequate for the system or a nonlinear controller is necessary according to the nonlinearity measures. In the past decades, researchers have made extensive studies on nonlinearity measures, and have proposed quite a few definitions and computational methods [1–13]. Most of them are defined as a distance between the nonlinear system and its best linear approximation [2–9]. Although the definitions are intuitive, the general computation of the best linear models and nonlinearity measures are rather complicated [2]. Besides, most of them cannot be used in feedback controller synthesis directly [3].

Recently, the gap metric which was recognized as being more appropriate to measure the distance between two linear systems than a norm-based metric [17,18], has been employed to quantify the nonlinearity level of industrial processes. And several definitions have been developed [13–16]. The nonlinearity measures based on the gap metric are comparatively easier and simpler to compute and apply. And some of them have been used for multi-model decomposition in the multi-model control framework [15,16].

The multi-model control approaches have been popular in controlling chemical processes with wide operating ranges and large set-point changes [19–37]. The key point is to decompose a nonlinear system into a set of linear subsystems, so that classical linear control strategies can be easily adopted. Generally, the multi-model control approaches comprise three elements: the multi-model decomposition (*i.e.*, model bank determination), the local controller design, and the local controller combination. From an integration perspective, it is necessary to connect the three elements closely, so that local model redundancy can be avoided to simplify the controller structure, dependency on previous knowledge can be reduced to make the design procedure more systematic, computational load can be decreased to make the method more efficient, and performance of the controller can be raised to make the method more effective. Therefore, integrated multi-model control methods have been recently put forward [19,28–30], in which the model bank determination, the local linear controller design, and the local

* Corresponding author.
E-mail addresses: hzdujing@163.com, du.jingjing.china@gmail.com (J. Du).

http://dx.doi.org/10.1016/j.jprocont.2017.07.001
0959-1524/

linear controller combination are fully or partly integrated. Two integrated multi-model control design frameworks were proposed in Ref. [19]. One method (Algorithm 2) uses the maximum stability margin (which is comparatively controller-independent) while the other (Algorithm 1) uses the actual stability margin of a given controller design. Although Algorithm 2 from Ref. [19] is simpler, it has a tuning parameter which depends on a priori knowledge. Algorithm 1 from Ref. [19] is more systematic; however, it is more complicated and involves intensive computation and tests. In Ref. [20], a weighting method with only one tuning parameter was proposed based on the gap metric, in which the weights can be computed off-line and kept in a look-up table. Here we call it 1-δ method for simplicity. It is intuitive and simple compared to traditional methods. Therefore, Ref. [29] used it to connect the local controller combination with the other two steps to propose an integrated multi-linear model predictive control method. However, the 1-δ method is not sensitive to the tuning parameter, which is undesirable.

In this paper, a control-relevant nonlinearity measure (CRNM) method is proposed to quantify the nonlinearity extent of a process based on the gap metric, which can be used directly in controller synthesis: It offers guidance for controller design; and it sets up a criterion to assess the controller's performance. The proposed CRNM method is then employed to perform model bank determination and local controller design in a multi-model control framework. Besides, a $1/\delta$ gap-based weighting method, which has all the advantages of the 1-δ method and is more sensitive to the tuning parameter, is put forward to combine the local controllers. Thus an improved integrated multi-model control framework is established based on CRNM, which integrates the advantages of the algorithms Ref. [19] while overcomes their disadvantages. The proposed integrated multi-model control approach aims to realize four goals. (a) To select as few linear models as necessary to design a multi-model controller, so that the model redundancy can be avoided; (b) to use as little a priori knowledge as possible, so that the method can be systematic and user-friendly; (c) to reduce computational load as much as possible, so that the method can be easy to implement; (d) to schedule the local controllers as well as possible so that the global multi-model controller can be more effective. Two CSTRs are simulated to illustrate the use of the improved integrated multi-model control approach. Simulation results demonstrate that the proposed CRNM-based integrated multi-model framework is systematic, efficient and effective, and performs better than related multi-model control methods [20,26].

This paper is organized as follows. Related background about the gap metric and the gap metric stability margin is shortly reviewed in Section 2. In Section 3, a control-relevant nonlinearity measure method is proposed. Supported by the proposed nonlinearity measure, an integrated multi-model control approach is proposed in Section 4, which includes a CRNM-based multi-model decomposition and local controller design procedure and a $1/\delta$ gap-based weighting method. Closed-loop simulations are present in Section 5 to illustrate the effectiveness of the proposed approaches, and comparisons have been made with related methods. In Section 6, some conclusions are made about the paper.

2. Gap metric and gap metric stability margin

Relevant background about the gap metric and the gap metric stability margin is briefly recalled in this section.

2.1. Gap metric

The gap metric between two linear systems P_1 and P_2 with their normalized right coprime factorizations $P_1 = N_1 M_1{}^{-1}$ and $P_2 = N_2 M_2{}^{-1}$, is denoted as $\delta(P_1, P_2)$ and is defined by the maximum of two directed gaps [17]:

$$\delta(P_1, P_2) := \max\{\vec{\delta}(P_1, P_2), \vec{\delta}(P_2, P_1)\} \tag{1}$$

$$\vec{\delta}(P_1, P_2) = \inf_{Q \in H_\infty} \left\| \begin{bmatrix} M_1 \\ N_1 \end{bmatrix} - \begin{bmatrix} M_2 \\ N_2 \end{bmatrix} Q \right\|_\infty \quad \text{where and .}$$

The gap metric between any two linear systems is bounded between 0 and 1. Therefore, the gap metric is more intuitive than a metric based on norms. Besides, the gap metric offers some useful information for control system analysis and synthesis. For example, if the gap metric between two systems is close to 0, then at least one feedback controller can be found to stabilize both of them; otherwise if the gap is close to 1, it will be difficult of impossible to design a feedback controller that can stabilize both systems [18].

2.2. Gap metric stability margin

Suppose K is a feedback controller that can stabilize the linear system P, then the gap metric stability margin of the closed-loop system is defined as [38]:

$$b_{P,K} = \left\|\begin{bmatrix} I \\ K \end{bmatrix}(I+PK)^{-1}[I \quad P]\right\|_\infty^{-1} = \left\|\begin{bmatrix} I \\ P \end{bmatrix}(I+KP)^{-1}[I \quad K]\right\|_\infty^{-1} \tag{2}$$

where I is the identity matrix. The gap metric stability margin is also called the normalized coprime stability margin.

Denote the left normalized coprime factors of P as $P = \tilde{M}^{-1}\tilde{N}$, and the Hankel norm as $\| \; \|H$. Then the maximum gap metric stability margin of P is defined as [38]:

$$\begin{aligned} b_{opt}(P) &:= \left\{ \inf_{K\,\text{stabilizing}} \left\|\begin{bmatrix} I \\ K \end{bmatrix}(I+PK)^{-1}[I \quad P]\right\|_\infty \right\}^{-1} \\ &= \sqrt{1 - \|[\tilde{N} \quad \tilde{M}]\|_H^2} < 1 \end{aligned} \tag{3}$$

From Eq. (3), it is clear that the maximum stability margin is an intrinsic property of the plant P, and has nothing to do with the controller. Besides, for the same system P, the maximum stability is greater than or equal to $b_{P,K}$ for any controller K.

The connection between the gap metric and the gap metric stability margin is shown by Proposition 1.

Proposition 1[38]. *Suppose the feedback system with the pair (P_0, K) is stable. Let $\mathcal{P} := \{P : \delta(P, P_0) < r\}$. Then the feedback system with the pair (P, K) is also stable for all $P \in \mathcal{P}$ if and only if*

$$b_{P_0,K} \geq r > \delta(P, P_0) \tag{4}$$

Once a nonlinear process is linearized around a set of equilibrium points, the gap metric and the gap metric stability margin are usable. In this work, we will use the gap metric and the gap metric stability margin to propose a CRNM method on the basis of Proposition 1. The gap metric and maximum stability margin of the system are used to define a preliminary nonlinearity measure NM_1 for guidance before a controller is designed, and afterwards the gap metric and actual stability margin of the closed-loop system are used to define a secondary nonlinearity measure NM_2 to qualify the performance of the controller. If NM_2 of the considered system is smaller than 1, it means that the linear controller is capable to stabilize it. Otherwise, we will decompose the nonlinear system into a set of linear subsystems and design a set of local linear controllers according to the nonlinearity measure criteria. Thus, the proposed CRNM method tells us whether the linear controller is capable to stabilize the nonlinear system or not.

Besides, the gap metric is also used for controller combination in the multi-model control of nonlinear systems by some researchers [20,21]. In section 4, this work will proposed a $1/\delta$ gap-based