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Identification of higher-dimensional ill-conditioned systems using extensions of virtual transfer function between inputs

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ABSTRACT

A new formulation is proposed to directly extend the virtual transfer function between inputs (VTFBI) approach to ill-conditioned systems with dimensions higher than 2×2. The method requires only a single correlated component and is applicable to moderately large systems of up to around six outputs. To cater for systems with even higher dimensions, an indirect approach is further introduced based on subsystem decomposition in which the design for each subsystem achieves D-optimality in the presence of active output variance constraints. New measures of sensitivity to measurement inaccuracy and parameter changes are also introduced. A detailed case study shows that both direct and indirect extensions of the VTFBI technique outperform competing ones in terms of accuracy in the estimation of singular values, robustness towards the effect of noise as well as effectiveness for application in model based control. An additional advantage of the proposed approaches is that their performance does not depend significantly on the specific design choices made within these methods.

1. Introduction

Processes encountered in the industry are often multivariable. Requirements for the identification of such systems are significantly different from those for single-input single-output systems. Accurate estimation of the individual transfer function elements in a transfer function matrix may not be sufficient to ensure robust closed loop stability [1]. For model-based control such as model predictive control (MPC), precise identification of the singular values is key to its successful implementation [2,3]. However, if the system is ill-conditioned, estimation of the smallest singular value is a challenging task which requires specially designed experiments. In addition, these experiments not only have to take into consideration operational constraints, but the costs incurred while doing so need to be minimized [4].

The identification methods in the literature can generally be classified into open loop methods [5–9], closed loop methods [10–13] or a combination of both [3]. Both open loop and closed loop approaches have their advantages and disadvantages. In general, the low gain direction is more easily excited in the closed loop [10]. However, experiments in the closed loop come with the added complexity of having to implement feedback controllers and to deal with the bias in the estimates caused by the feedback [11,14]. On the other hand, open loop approaches are free from the complexities resulting from the feedback but the perturbation signals must be designed carefully to sufficiently excite the low gain direction. It can be seen from [1] that both open loop and closed loop techniques seem equally popular in commercial software.

Due to the different challenges, open loop techniques require extra care in the signal design whereas closed loop ones necessitate more effort in the estimation. The focus of the current paper is on open loop methods. In [5], low amplitude uncorrelated binary signals were used to perturb the high gain direction while high amplitude correlated binary signals were utilized to excite the low gain direction. A similar idea was proposed in the frequency domain using a modified “zippered” power spectrum [6] consisting of alternating correlated harmonics with high levels of power and uncorrelated harmonics with lower levels of power. The design is realized using multisine signals. Another technique makes use of “rotated inputs” where the rotation can be applied to binary signals as well as multisine signals [7,15]. The technique is based on a geometric approach. Recently, an open loop method based on virtual transfer function between inputs (VTFBI) [9] was introduced, with a geometric interpretation for 2×2 systems. The method uses a correlated harmonic to increase the excitation in the low gain

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direction in combination with uncorrelated harmonics which allow the contributions of the different inputs to be easily decoupled. The technique was shown to be able to achieve high estimation accuracy due to improved distribution of power to uncorrelated harmonics as well as robustness to changes in the gain directions with frequency.

In the current paper, the extension of the VTFBI technique to higher-dimensional systems is investigated, noting that most studies in the literature on ill-conditioned systems are limited to 2×2 systems [16]. The main contributions are (i) a formulation of the direct extension of the VTFBI approach to higher-dimensional systems for moderately large systems with an advantage of requiring only a single correlated harmonic, (ii) an indirect extension applicable to higher-dimensional systems utilizing a larger number of correlated harmonics designed to achieve D-optimality, (iii) new measures of sensitivity to measurement inaccuracy and parameter changes, (iv) a detailed case study presenting for the first time a comparison of the VTFBI method with the modified zippered spectrum approach (which is at present one of the most successful methods [17]) and the rotated inputs approach on a higher-dimensional system, and (v) in-depth analysis of the effect of different choices of subsystem pairing related to the indirect extension.

The rest of the paper is organized as follows. Section 2 provides a brief introduction of the VTFBI technique. Direct and indirect extensions of the VTFBI approach are discussed in Section 3. New sensitivity measures are proposed in Section 4. A detailed case study on a 3×3 system is described in Section 5. Finally, concluding remarks are drawn in Section 6.

2. VTFBI technique

Consider a multivariable system with n inputs $u_1, u_2, \ldots, u_n$ and n outputs $y_1, y_2, \ldots, y_n$ defined by

$$\mathbf{Y}(\mathbf{s}) = \mathbf{G}(\mathbf{s})\mathbf{U}(\mathbf{s}),\ \mathbf{G}(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & \ldots & G_{1n}(s) \\ G_{21}(s) & G_{22}(s) & \ldots & G_{2n}(s) \\ \vdots & \vdots & & \vdots \\ G_{n1}(s) & G_{n2}(s) & \ldots & G_{nn}(s) \end{bmatrix} \tag{1}$$

where $\mathbf{U}(s) = [\,U_1(s) \quad U_2(s) \quad \ldots \quad U_n(s)\,]^T$ and $\mathbf{Y}(s) = \left[\,Y_1(s) \quad Y_2(s) \quad \ldots \quad Y_n(s)\,\right]^T$ are vectors of the Laplace transform of the inputs and outputs, respectively, and $\mathbf{G}(s)$ is the transfer function matrix. It is assumed that at least one element in each row and column of $\mathbf{G}(s)$ is nonzero. Applying singular value decomposition

$$\mathbf{G}(s) = \left[\,\mathbf{b}_1(s) \quad \mathbf{b}_2(s) \quad \ldots \quad \mathbf{b}_n(s)\,\right] \times \begin{bmatrix} \sigma_1(s) & 0 & \ldots & 0 \\ 0 & \sigma_2(s) & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots & \sigma_n(s) \end{bmatrix} \begin{bmatrix} \mathbf{a}_1^T(s) \\ \mathbf{a}_2^T(s) \\ \vdots \\ \mathbf{a}_n^T(s) \end{bmatrix} \tag{2}$$

where $\mathbf{a}(s)$ and $\mathbf{b}(s)$ are the singular vectors of the input and output, respectively, and the singular values are arranged such that $\sigma_1(s) \geq \sigma_2(s) \geq \ldots \geq \sigma_n(s)$. High and low gain directions refer to singular vectors corresponding to the maximum and minimum singular values, respectively.

In the VTFBI technique proposed for a 2×2 system with transfer function matrix $\mathbf{G}(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$, the idea is to improve uniformity in the output state-space so that the low gain direction is sufficiently excited. The geometric approach aims to align a single correlated component of the input, selected at a frequency ω_0, to the direction where the output trajectory in the state-space traces a circle of radius r, such that

$$y_1(t) = r\sin(\omega_0 t), \tag{3A}$$

$$y_2(t) = \pm r\cos(\omega_0 t). \tag{3B}$$

By taking the Laplace transform of (3A) and (3B), the relationship between u_1 and u_2 at ω_0 can be derived and this is defined as the VTFBI $H(s)$ where [9]

$$H(s) = \frac{U_2(s)}{U_1(s)} = \frac{-G_{21}(s)Y_1(s) + G_{11}(s)Y_2(s)}{G_{22}(s)Y_1(s) - G_{12}(s)Y_2(s)} = \frac{\pm sG_{11}(s) - \omega_0 G_{21}(s)}{\mp sG_{12}(s) + \omega_0 G_{22}(s)}. \tag{4}$$

Besides the correlated component, the input signals have uncorrelated components covering the frequency range of interest. These enable easy decoupling of the contributions of the different inputs and serve to ensure the accuracy of the individual transfer function estimates. Thus with the VTFBI design, both the issues of gain directionality and model mismatch are taken care of. The uncorrelated components are implemented using persistently exciting multisines to ensure identifiability. The number of excited uncorrelated harmonics in each signal must be at least equal to the maximum order of the transfer functions perturbed by the signals [14]. This requirement is usually not difficult to satisfy in practice.

3. Extension to higher-dimensional systems

3.1. Direct extension

For an $n \times n$ system with $n > 2$, the direct extension aims to achieve output trajectories as close as possible to circles when viewed from two-dimensional planes, using only a single correlated component of the input. The optimization problem is formulated as

$$\boldsymbol{\psi}_{\text{opt}} = \arg\min_{\boldsymbol{\psi}} \left[\sum_{\Pi_{ij}} \min\left\{ \left(\|\psi_i - \psi_j\| - \frac{\pi}{2}\right)^2, \left(\|\psi_i - \psi_j\| - \frac{3\pi}{2}\right)^2 \right\} \right] \tag{5}$$

where $\boldsymbol{\psi} = [\,\psi_1 \quad \psi_2 \quad \ldots \quad \psi_n\,]$ and Π_{ij} denotes all combinations of i and j, for $i, j = 1, 2, \ldots, n$ and $i \neq j$. In (5), $0 \leq \psi_i < 2\pi$; however, for the sake of compactness in the subsequent parts of the paper, the phases are sometimes written outside this range but this should not cause any confusion since it is understood that the phases are cyclic with a period of 2π. Without loss of generality, $\psi_1 = 0$.

For $n = 3$, two solutions exist where the phases are equally spaced, corresponding to $\boldsymbol{\psi}_{\text{opt}} = \left[\,0 \quad \pm 2\pi/3 \quad \pm 4\pi/3\,\right]$. The outputs can be expressed as

$$y_1(t) = r\sin(\omega_0 t) \quad \Rightarrow Y_1(s) = \frac{r\omega_0}{s^2 + \omega_0^2}, \tag{6A}$$

$$y_2(t) = r\left(-\frac{1}{2}\sin(\omega_0 t) \pm \frac{\sqrt{3}}{2}\cos(\omega_0 t)\right) \Rightarrow Y_2(s) = r\left(\frac{-\omega_0 \pm \sqrt{3}s}{2\left(s^2 + \omega_0^2\right)}\right), \tag{6B}$$

$$y_3(t) = r\left(-\frac{1}{2}\sin(\omega_0 t) \mp \frac{\sqrt{3}}{2}\cos(\omega_0 t)\right) \Rightarrow Y_3(s) = r\left(\frac{-\omega_0 \mp \sqrt{3}s}{2\left(s^2 + \omega_0^2\right)}\right). \tag{6C}$$

Writing the inputs in the form $U_i(s) = r\left(\frac{c_i\omega_0 + d_i s}{s^2 + \omega_0^2}\right)$; $i = 1, 2, 3$; the values of $\mathbf{d}_i = [\,c_i \quad d_i\,]^T$ which specify the input amplitudes and