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Unsteady flow and heat transfer of power-law nanofluid thin film over a stretching sheet with variable magnetic field and power-law velocity slip effect

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ABSTRACT

This paper studies the flow and heat transfer of the power-law nanofluid thin film due to a stretching sheet with magnetic field and velocity slip effects. Unlike classical work, Fourier's law is modified by assuming that the thermal conductivity is power-law-dependent on the velocity gradient. Meanwhile, the power law wall temperature and the power law velocity slip effects are taken into account. Three different types of nanoparticles, Al_2O_3, TiO_2 and CuO are considered with ethylene vinyl acetate copolymer (EVA) used as a base fluid. The governing equations are solved by using DTM–NIM which is combined the differential transform method (DTM) with Newton Iteration method (NIM). The results show that for the specific physical parameters, the two different velocity profiles have always an intersection which goes from a far-field region to the stretching sheet as increasing velocity slip parameter. Furthermore, CuO–EVA nanofluid has better enhancement on heat transfer than TiO_2/Al_2O_3–EVA.

1. Introduction

The development trend of functional thin film is paid more and more attentions, because it has extensive application in daily life, industry and technology field like food packing, solar cell encapsulating material and anti-counterfeiting film, *etc*. In recent years, the rheological properties of thin film made by tape casting have attracted the attention of many scholars. Wang [1] firstly studied flow of liquid film on an unsteady stretching surface in hydrodynamics. And then Wang [2] obtained the analytical solutions. Subsequently, Liu and Andersson [3] analyzed the heat transfer in a liquid film. The flow and heat transfer of thin film over an unsteady stretching sheet were extended by Noor and Hashim [4], in which the boundary layer velocity and temperature are functions of the position and time. Further, Marangoni convection induced by temperature gradient on the interface was considered by Noor. They found heat diffusivity near the stretching sheet was enhanced for the larger thermocapillarity number. Khan et al. [5] investigated the effects of variable viscosity on a thin film flow over a shrinking/stretching sheet. Other references concerning this field contain variable thermal conditions [6], heat transfer [7], magnetic field [8], first-order chemical reaction [9] *etc.*

The majorities of the above researches focus on the Newtonian fluids, however, the polymer melts or solutions as materials in form of thin film are no longer Newtonian fluids. A significant attention has been devoted to describe the behavior of non-Newtonian fluids. A number of models have been proposed to capture the rheology of non-Newtonian fluids, in which the shear stress is power-law function of the strain rate. EVA is a copolymer of ethylene and vinyl acetate, which is used in encapsulation of photovoltaic module [10]. The rheological behavior of EVA nanocomposites was analyzed by Hwang et al. [11], in which presented the EVA nanocomposites displayed shear thinning characteristic. The result showed that it was a typical pseudo-plastic (power-law) fluid.

Currently, many scholars have done lots of studies and made great achievements on the power-law fluids. Andersson et al. [12] and Chen [13] considered the power law viscosity just in momentum equation. Pop et al. [14] proposed the modified model in which thermal conductivity depended on the power exponent of velocity gradient. Zheng et al. [15] introduced the power law viscosity in energy equation, and analyzed the heat transfer driven by power-law temperature gradient. Wang and Pop [16] discussed the flow of a power law fluid film on an unsteady stretching sheet. Zhang et al. [17] considered the Marangoni convection of power-law fluids with linear temperature distribution. Aziz et al. [18] explored the slip effect on a power-law fluid past a porous flat plate embedded in the Darcy type porous medium. Jiao et al. [19] proposed the heat and mass transfer constitutive equa-

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tion based on *N*-diffusion proposed by Philip and the abnormal convection–diffusion proposed by Pascal. A mathematical model of MHD boundary layer flow and heat transfer past a nonlinear vertically porous stretching sheet with the effects of hydrodynamic and thermal slip was established by Alkahtani et al. [20]. Si et al. [21] investigated the laminar film condensation of pseudo-plastic non-Newtonian fluids with variable thermal conductivity on an isothermal vertical plate. Silva et al. [22] presented a new mathematical model about a power-law fluid flowing in a channel partially filled with a homogeneous and isotropic porous medium, and obtained numerical results. Further, the unsteady boundary layer flow and heat transfer of power law fluid model over a radially stretching sheet was extended by Ahmed et al. [23] and new nonlinear diffusion model about laminar boundary layer flow of power law fluid was introduced by Lin et al. [24].

Because of unexpected thermal properties, in recent years, nanofluids have attracted great attention in heat transfer enhancement of the renewable energy systems, industrial thermal management and material processing. Nanofluid is generated by mixing the nano-scale solid particles into the base fluid [25]. As a copolymer, EVA has poor thermal conductivity. To achieve its heat transfer efficiency, the metal or metallic oxide nanoparticles are mixed in it. Yuwawech et al. [26] proved that the better mechanical and barrier properties were obtained when the modified bacterial cellulose nanofibers were added in the EVA film. The flow and heat transfer of a nano-liquid over an unsteady stretching surface was analyzed by Xu et al. [27]. Lin et al. [28] explored the viscous dissipation effects on the heat transfer of pseudo-plastic nanoliquid in a finite thin film. Zhang et al. [29] examined the flow and heat transfer of an Oldroyd-B nanofluid thin film. Many researchers studied this problem in following aspects such as exponentially stretching sheet [30], thermal radiation effect [31], internal heat generation [32] and three dimensional boundary layer flow [33] *etc.*

All of the above mentioned investigations are restricted to the flow with no slippage condition at the surface. However, it is significant that the velocity slip occurs in polymer melts. The hydrodynamic slip flow has abundant applications in the field of engineering. Mahmoud [34] considered slip velocity effect on a non-Newtonian power-law fluid over a moving permeable surface. Nandy [35] discussed unsteady flow of Maxwell fluid in the presence of nanoparticles with Navier slip. Dhanai et al. [36] dealt with the numerical investigation of non-Newtonian nanofluid flow with slip, heat source/sink and variable magnetic field.

The similarity solution is an important way to analyze the flow and heat transfer of the thin film. The method of DTM is used widely to get the similarity solution for its advantage of no need restrictive assumptions or linearization. It was introduced by Zhou [37]. Then, using DTM, Zhang et al. [38] solved the MHD flow and radiation heat transfer of nanofluids in porous media with variable surface heat flux and chemical reaction. Zheng and Su [39] proposed the DTM-BF method which modified DTM by choosing the flexible base function, to solve the differential equations of unbounded domains more effectively. Using this method, Rashidi and Erfani [40] and Su et al. [41] solved the similar problem of nano boundary layer and MHD flow over permeable shrinking sheet respectively.

On the basis of previous work, the rheological characteristics of melted EVA thin film is investigated in this paper. The melted EVA (28% VAC) is used as the base fluid and three types of nanoparticles are considered. The unsteady boundary layer flow and heat transfer of an incompressible power-law nanofluid thin film due to a stretching sheet with the power-law wall temperature and power-law velocity slip effect have been investigated analytically. The effect of power-law viscosity on temperature field is taken into account with the modified Fourier's law. The governing PDEs are reduced to ODEs by the flexible similarity transformations and then solved analytically by the DTM–NIM method based on differential transform method (DTM) and Newton Iteration method (NIM). Moreover, the effects of relevant parameters on the velocity and temperature fields are shown graphically.

2. Mathematical formulation

Consider the unsteady two-dimensional flow of an incompressible pseudo-plastic EVA nanofluid in a finite film over a stretching sheet. A schematic of the physical model and Cartesian coordinate system is shown in Fig. 1. The melted EVA (28% VAC) is the base fluid. The nanoparticle contains three different types such as Al_2O_3, TiO_2 and CuO. The sheet velocity and temperature both vary with time and space. The base fluid and the nanoparticles are assumed to be in thermal equilibrium and the flow is laminar. Under these assumptions, the conservation equations of mass, momentum and energy for the unsteady state flow of power-law fluid boundary layer can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon_{nf}\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) - \frac{\hat{\sigma}B^2}{\rho_{nf}}u, \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial T}{\partial y}\right), \tag{3}$$

with boundary conditions

$$u{=}U_w + \lambda_1\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right),\ v = 0,\ T = T_w,\ y = 0 \tag{4}$$

$$\frac{\partial u}{\partial y} = 0,\ v = u\frac{\partial h}{\partial x} + \frac{\partial h}{\partial t},\ \frac{\partial T}{\partial y} = 0,\ y = h(x,t) \tag{5}$$

where u and v are the velocity components in the x- and y-directions, respectively. t is the time, $\hat{\sigma}$ is the electrical conductivity and T indicates the temperature. n is the power law index. A uniform magnetic field is assumed to exist in the direction normal to the surface $B = B_0/(1-\alpha t)^{1/2}$, where α is a positive constant. $\upsilon_{nf} = \mu_{nf}/\rho_{nf}$ is the kinematic viscosity of nanofluid, in which μ_{nf} is the dynamic viscosity of nanofluid and ρ_{nf} is the density of nanofluid. $\alpha_{nf} = k_{nf}/(\rho C_p)_{nf}$ represents the thermal diffusivity of the nanofluid, and k_{nf}, $(\rho C_p)_{nf}$ represent the thermal conductivity and the heat capacitance of the nanofluid respectively. Subscript *nf* indicates the nanofluid property.

In boundary conditions, λ_1 is the slip coefficient, the effect of power-law slip is considered [34,36,42]. And $h(x,\ t)$ is the uniform thin film thickness. U_w and T_w are the velocity and temperature of the sheet, respectively. The stretching velocity is assumed as

$$U_w = \frac{bx}{1-\alpha t}, \tag{6}$$

where b is a positive constant and denotes the initial stretching rate. The effective stretching rate is increasing with time when $t < 1/\alpha$.

Further, the temperature of the elastic sheet is assumed to vary both along the sheet and with time,

$$T_w = T_0 - T_{ref}\frac{b^{2-n}}{\upsilon_f}x^{3-\frac{2}{n+1}}(1-\alpha t)^{-3+\frac{3}{n+1}}, \tag{7}$$

where T_0 is the temperature at the wall, T_{ref} is a constant in the case of $t < 1/\alpha$, while the special case is $n = 1$, Eq. (7) is simplified as $bx^2(1-at)^{-3/2}$ which has been studied by Wang and Pop [16].