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Brief paper

Solvability conditions and design for state synchronization of multi-agent systems☆

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ABSTRACT

This paper derives conditions on the agents for the existence of a protocol which achieves synchronization of homogeneous multi-agent systems (MAS) with partial-state coupling, where the communication network is directed and weighted. These solvability conditions are necessary and sufficient for single-input agents and sufficient for multi-input agents. The solvability conditions reveal that the synchronization problem is primarily solvable for two classes of agents. This first class consists of at most weakly unstable agents (i.e. agents have all eigenvalues in the closed left half plane) and the second class consists of at most weakly non-minimum-phase agents (i.e. agents have all zeros in the closed left half plane). Under our solvability condition, we provide in this paper a design, utilizing H_∞ optimal control.

1. Introduction

Many researchers have been working on the synchronization problem of multi-agent systems (MAS) over the last decade. The area has spread from theoretical research to different applications, such as robot networks, sensor networks, power networks, social networks, and so on. The goal of synchronization is to secure asymptotic agreement on a common state (*state synchronization*) or output trajectory (*output synchronization*) among agents of the network through decentralized control protocols. Early work can, for instance, be found in Olfati-Saber and Murray (2004), Ren and Atkins (2007), Tuna (2008), Wu and Chua (1995) and Yang, Roy, Wan, and Saberi (2011) for the state synchronization that requires homogeneous MAS (i.e. agents are identical), and in Bai, Arcak, and Wen (2011), Chopra and Spong (2008a), Kim, Shim, and Seo (2011), Wieland, Sepulchre, and Allgöwer (2011), Xiang and Chen (2007) and Zhao, Hill, and Liu (2010) for the output synchronization of heterogeneous MAS.

For state synchronization, two kinds of communication networks are considered, one pertaining to full-state coupling and the other to partial-state coupling. We consider purely decentralized protocols which are non-introspective and do not share protocol states over the network. For MAS with full-state coupling, there is no restriction on the agent model. That is, synchronization can be always be achieved for stabilizable agents via purely decentralized protocols (e.g., Wang, Cheng, and Hu, 2008; Wieland, Kim, Scheu, and Allgöwer, 2008). For MAS with partial-state coupling, work has focused on conditions to achieve consensus by using static output-feedback protocols, for example (Ma & Zhang, 2010; Tuna, 2009; Xia & Scardovi, 2016a, 2016b). It is shown that only a restricted class of agents satisfy these solvability conditions. When using dynamic output-feedback protocols, synchronization can be achieved for a larger class of agents. It is well-known in the literature that the synchronization problem can be solved via solving a simultaneous stabilization problem. The solvability of these simultaneous stabilization problems requires a specific and stringent requirement on the right half plane (RHP) poles and zeros of the agent model, that is the simultaneous presence of RHP poles and zeros is not allowed. Therefore, in the literature on MAS with partial-state coupling and purely decentralized protocols, three kinds of restrictions can be observed.

Agents are at most weakly unstable In Seo, Shim, and Back (2009), general multi-input multi-output (MIMO) agents with all eigenvalues in the closed-left half plane are considered and a low-gain based protocol is proposed. This work is extended in Wang, Saberi, Stoorvogel, Grip, and Yang (2013) to tolerate time delay in the input for each agent. In Yang, Saberi, Stoorvogel, and Grip (2014), pre-compensators are developed to shape non-identical agents to almost identical agents with all eigenvalues in the closed-left half plane.

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Agents are at most weakly non-minimum-phase More specifically, Grip, Saberi, and Stoorvogel (2015) and Zhang, Saberi, Grip, and Stoorvogel (2015) require the agents to be minimum-phase. Chopra and Spong (2008a) require agents to be weakly minimum-phase, and Stoorvogel, Zhang, Saberi, and Grip (2014) and Stoorvogel, Saberi, and Zhang (2016) require agents to be weakly non-minimum-phase with the restriction that the Jordan blocks associated to zeros on the imaginary axis are at most size 2.

Agents are passive with positive definite storage function This implies, among other conditions, that the agents are neutrally stable and weakly minimum-phase. See Arcak (2007), Bai et al. (2011), Chopra and Spong (2008b), Yao, Guan, and Hill (2009) and Zhao et al. (2010) and references therein.

In this paper our contribution is twofold.

- For agents with single input, we give a necessary and sufficient condition for the solvability of state synchronization of a MAS.
- For agents with multiple inputs, we give a sufficient condition for the solvability of state synchronization of a MAS. The sufficient condition identifies two major classes of agents, for which the synchronization problem can be solved: at most weakly unstable agents (poles in the closed left half plane) and at most weakly non-minimum-phase agents (zeros in the closed left half plane).

Under the sufficient condition we provide a protocol design that utilizes H_∞ optimal control.

1.1. Notations and definitions

Given a matrix $A \in \mathbb{C}^{m\times n}$, A' denotes its conjugate transpose, $\|A\|$ is the induced 2-norm. A square matrix A is said to be Hurwitz stable if all its eigenvalues are in the open left half complex plane. We denote by col$\{x_1, \ldots, x_N\}$, a column vector with $x_1, \ldots, x_N$ stacked together. $A \otimes B$ depicts the Kronecker product between A and B. I denotes the identity matrix and 0 denotes the zero matrix with their dimensions clear from the context.

A *weighted directed graph* $\mathcal{G}$ is defined by a triple $(\mathcal{V}, \mathcal{E}, \mathcal{A})$ where $\mathcal{V} = \{1, \ldots, N\}$ is a node set, $\mathcal{E}$ is a set of pairs of nodes indicating connections among nodes, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the weighting matrix, and $a_{ij} > 0$ iff $(i, j) \in \mathcal{E}$. Each pair in $\mathcal{E}$ is called an *edge*. A *path* from node i_1 to i_k is a sequence of nodes $\{i_1, \ldots, i_k\}$ such that $(i_j, i_{j+1}) \in \mathcal{E}$ for $j = 1, \ldots, k-1$. A *directed tree* with *root* r is a subset of nodes of the graph $\mathcal{G}$ such that a path exists between r and every other node in this subset. A *directed spanning tree* is a directed tree containing all the nodes of the graph. The set of all root agents for a graph $\mathcal{G}$ is denoted by $\Pi_{\mathcal{G}}$.

For a weighted graph $\mathcal{G}$, a matrix $L = [\ell_{ij}]$ with

$$\ell_{ij} = \begin{cases} \sum_{k=1}^{N} a_{ik}, & i = j, \\ -a_{ij}, & i \neq j, \end{cases}$$

is called the *Laplacian matrix* associated with the graph $\mathcal{G}$. In the case where $\mathcal{G}$ has non-negative weights, L has all its eigenvalues in the closed right half plane and at least one eigenvalue at zero associated with right eigenvector $\mathbf{1}$.

The remainder of the paper is organized as follows. Section 2 describes the synchronization problem for MAS with partial-state coupling to be solved in this paper. Section 3 provides some preliminaries. Section 4 develops the solvability condition for the synchronization problem.

2. Problem formulation

Consider a MAS composed of N identical linear time-invariant continuous-time agents of the form,

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i, \\ y_i &= Cx_i, \end{aligned} \qquad (i = 1, \ldots, N) \tag{1}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$, $y_i \in \mathbb{R}^p$ are respectively the state, input, and output vectors of agent i.

The communication network provides each agent with a linear combination of its own output relative to that of other neighboring agents. In particular, each agent $i \in \{1, \ldots, N\}$ has access to the quantity,

$$\zeta_i = \sum_{j=1}^{N} a_{ij}(y_i - y_j) = \sum_{j=1}^{N} \ell_{ij} y_j, \tag{2}$$

where $[a_{ij}]$ is the weighting matrix and $[\ell_{ij}]$ is the Laplacian matrix of the associated graph $\mathcal{G}$ with nodes corresponding to the agents in the network.

Definition 1. *Let $\mathbb{G}^N$ denote the set of directed graphs with N nodes that contain a directed spanning tree. For any given $\alpha \geq \beta > 0$, the subset $\mathbb{G}^N_{\alpha,\beta} \subset \mathbb{G}^N$ requires additionally that the corresponding Laplacian matrix has the property that its nonzero eigenvalues have a real part larger than or equal to β and have an amplitude less than α.*

According to Ren and Beard (2005, Lemma 3.3), the Laplacian matrix L associated to any graph $\mathcal{G} \in \mathbb{G}^N_{\alpha,\beta}$ has a simple eigenvalue at the origin, with corresponding right eigenvector $\mathbf{1}$. Let $\lambda_1, \ldots, \lambda_N$ denote the eigenvalues of L such that $\lambda_1 = 0$ and $\mathrm{Re}(\lambda_i) > 0, i = 2, \ldots, N$.

Since all the agents in the network are identical, we pursue state synchronization among agents. That is, we require

$$\lim_{t\to\infty} (x_i(t) - x_j(t)) = 0, \qquad \forall i, j \in \{1, \ldots, N\}. \tag{3}$$

We formulate the state synchronization problem for a network with identical agents as follows.

Problem 1. Consider a MAS described by (1) and (2). Let $\mathbf{G}$ be a given set of directed graphs such that $\mathbf{G} \subseteq \mathbb{G}^N$. The *state synchronization* problem with a set of network graphs $\mathbf{G}$ is to find, if possible, a linear time-invariant dynamic protocol of the form,

$$\begin{cases} \dot{\chi}_i = A_c\chi_i + B_c\zeta_i, \\ u_i = C_c\chi_i + D_c\zeta_i, \end{cases} \tag{4}$$

for $i = 1, \ldots, N$ where $\chi_i \in \mathbb{R}^{n_c}$, such that, for any graph $\mathcal{G} \in \mathbf{G}$ and for all initial conditions for the agents and their protocol, state synchronization among agents is achieved.

3. Preliminaries

As shown in Seo et al. (2009) and Yang et al. (2014), the state synchronization among agents in the network with partial-state coupling can be solved by equivalently solving a robust stabilization problem. The closed-loop of agent (1) and protocol (4) can be written as

$$\begin{cases} \dot{\bar{x}}_i = \begin{pmatrix} A & BC_c \\ 0 & A_c \end{pmatrix} \bar{x}_i + \begin{pmatrix} BD_c \\ B_c \end{pmatrix} \zeta_i, \\ y_i = \begin{pmatrix} C & 0 \end{pmatrix} \bar{x}_i, \end{cases} \tag{5}$$

for $i = 1, \ldots, N$ where

$$\bar{A} = \begin{pmatrix} A & BC_c \\ 0 & A_c \end{pmatrix}, \quad \bar{B} = \begin{pmatrix} BD_c \\ B_c \end{pmatrix}, \quad \bar{C} = \begin{pmatrix} C & 0 \end{pmatrix} \text{ and } \bar{x}_i = \begin{pmatrix} x_i \\ \chi_i \end{pmatrix}.$$