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Technical communique

Performance evaluation of sampled-data control of Markov jump linear systems[☆]

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ABSTRACT

This technical communique extends the recent results of Geromel and Gabriel (2015) to $\mathcal{H}_\infty$ sampled-data control design of Markov jump linear systems (MJLS). It fulfills a lack of a specific necessary and sufficient result in the literature of sampled-data control of MJLS in the context of $\mathcal{H}_\infty$ performance. Mean square stabilizability and performance determination are addressed and discussed in a unified theoretical viewpoint. As a natural consequence, it is shown that the previous result of Geromel and Gabriel (2015) is obtained as a particular case. A globally uniformly convergent algorithm is proposed to solve the design conditions. The theory is illustrated by means of an example.

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1. Introduction

Sampled-data is becoming more and more important in the area of deterministic dynamic system control design. It is mostly employed in modern control structures like digital control (in a general framework, see Chen and Francis (1995), Ichikawa and Katayama (2001) and Levis, Schluete, and Athans (1971) and the references therein). For stochastic systems, however, sampled-data control design did not receive as much attention as it deserves keeping in mind the theoretical and practical importance of Markov jump linear systems, see Boukas (2006), Hu, Shi, and Frank (2006) and Mao (2013). To fill this gap the aforementioned class of sampled-data control design is addressed and the main result of Geromel and Gabriel (2015) is generalized to cope with $\mathcal{H}_\infty$ performance. The independence (in probabilistic terms) of the sampling and parameter jump instants is a realist assumption that makes the problem simple to solve. However, determining a solution to the optimality conditions (if any) is not a simple task and requires, in general, the adoption of an interactive procedure similar to the one proposed in Geromel and Gabriel (2015) for the $\mathcal{H}_2$ case. A global monotone convergent algorithm able to treat $\mathcal{H}_\infty$ and $\mathcal{H}_2$ problems in a unified manner is proposed.

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This technical communique provides a necessary and sufficient condition that extends the results of Geromel and Gabriel (2015) in which optimal sampled-data control of MJLS is treated by a stochastic hybrid linear system approach. See also Ichikawa and Katayama (2001) for a collection of useful results in the deterministic context. The set of natural and real numbers are denoted by $\mathbb{N}$ and $\mathbb{R}$, respectively.

2. Problem statement

Consider a MJLS with state space realization

$$\dot{x}(t) = A_{\theta(t)}x(t) + B_{\theta(t)}u(t) + E_{\theta(t)}w(t), \tag{1}$$

$$z(t) = C_{\theta(t)}x(t) + D_{\theta(t)}u(t), \tag{2}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $w \in \mathbb{R}^r$, and $z \in \mathbb{R}^s$ are the state, the control, the exogenous input, and the controlled output, respectively. We denote $\theta(t) \in \mathbb{K} = \{1, 2, \ldots, N\}$, a time-varying function governed by a continuous-time Markov process with transition rate matrix $\{\lambda_{ij}\} = \Lambda \in \mathbb{R}^{N\times N}$. It is assumed that the system evolves from initial conditions $x(0) = 0$ and $\theta(0) = \theta_0$, with initial probability $\mathbb{P}(\theta_0 = i) = \pi_{i0}$, $\forall i \in \mathbb{K}$. As usual in the context of $\mathcal{H}_\infty$ performance, the exogenous input $w \in \mathcal{L}_2(\Omega, \mathcal{F}, \mathbb{P})$, or just $\mathcal{L}_2$, is a norm bounded random process, that is $\|w\|_2^2 = \int_0^\infty \mathcal{E}\{w(t)'w(t)\}dt < \infty$, where $(\Omega, \mathcal{F}, \mathbb{P})$ is a complete probability space equipped with a filtration $\mathcal{F}_t$, with $t \in \mathbb{R}_+$, see Costa, Fragoso, and Todorov (2013). The sequence $\{t_k\}_{k\in\mathbb{N}}$ is defined by successive sampling instants such that $t_0 = 0$ and $t_{k+1} - t_k = T > 0$, $\forall k \in \mathbb{N}$. The class of admissible control signals is characterized by a state feedback sampled-data linear control of the form $u(t) = L_{\theta(t_k)}x(t_k)$, $\forall t \in [t_k, t_{k+1})$, where matrices $\{L_i\}_{i\in\mathbb{K}}$ are the state feedback gains.

System (1)–(2) is reformulated as an alternative but equivalent hybrid system with special matrix structures

$$\dot{\xi}(t) = \begin{bmatrix} A_{\theta(t)} & B_{\theta(t)} \\ 0 & 0 \end{bmatrix}\xi(t) + \begin{bmatrix} E_{\theta(t)} \\ 0 \end{bmatrix} w(t), \tag{3}$$

$$z(t) = \begin{bmatrix} C_{\theta(t)} & D_{\theta(t)} \end{bmatrix}\xi(t), \tag{4}$$

$$\xi(t_k) = \begin{bmatrix} I & 0 \\ L_{\theta(t_k)} & 0 \end{bmatrix}\xi(t_k^-) \tag{5}$$

subject to the initial conditions $\xi(0^-) = \xi_0 = 0$ and $\theta(0^-) = \theta(0) = \theta_0$ and valid $\forall t \in [t_k, t_{k+1})$, $\forall k \in \mathbb{N}$. It is called Hybrid Markov Jump Linear System (HMJLS) and it is related to the closed-loop version of (1)–(2) through $\xi(t)' = [x(t)'\ u(t)']$, see Geromel and Gabriel (2015) for details. It is of great interest to solve the optimal control problem

$$\inf_{L_1,\ldots,L_N} \left\{ \gamma^2 \ :\ \sup_{w\in\mathcal{L}_2} \|z\|_2^2 - \gamma^2 \|w\|_2^2 \le 0 \right\}, \tag{6}$$

whose solution, in the present context, is not available in the literature to the best of the authors' knowledge.

3. Hybrid MJLS

Consider the following Hybrid MJLS

$$\dot{\xi}(t) = F_{\theta(t)}\xi(t) + J_{\theta(t)}w(t), \tag{7}$$

$$z(t) = G_{\theta(t)}\xi(t), \tag{8}$$

$$\xi(t_k) = H_{\theta(t_k)}\xi(t_k^-), \tag{9}$$

evolving from arbitrary initial conditions $\xi(0^-) = \xi_0$ and $\theta(0^-) = \theta(0) = \theta_0$. Clearly this model contains, as a particular case, the one given in (3)–(5) with special structured matrices. Consider the coupled differential Riccati equations

$$\dot{P}_i + F_i'P_i + P_iF_i + \gamma^{-2}P_iJ_iJ_i'P_i + \sum_{j\in\mathbb{K}} \lambda_{ij}P_j = -G_i'G_i,\ i \in \mathbb{K}, \tag{10}$$

whose solution, whenever exists, is unique, bounded and satisfies $P_i(t) \ge 0$, $\forall t \in [0, T]$ and $T > 0$, provided that $P_i(T) \ge 0$, $\forall i \in \mathbb{K}$, see Costa et al. (2013). For $\gamma > 0$ big enough, Eq. (10) collapses to the coupled differential Lyapunov equations

$$\dot{P}_i + F_i'P_i + P_iF_i + \sum_{j\in\mathbb{K}} \lambda_{ij}P_j = -G_i'G_i,\ i \in \mathbb{K}. \tag{11}$$

which always admits a bounded solution due to its linearity, (Costa et al., 2013). Thus, for a given $\gamma > 0$ our main purpose is to evaluate the function

$$\rho_\gamma(\xi_0) = \sup_{w\in\mathcal{L}_2} \|z\|_2^2 - \gamma^2 \|w\|_2^2 \tag{12}$$

which is bounded whenever the system is mean square stable. The constraint in problem (6) can be alternatively expressed as $\rho_\gamma(0) \le 0$ and the optimal value of $\rho_\infty(\xi_0) = \|z\|_2^2$ follows from $w \equiv 0 \in \mathcal{L}_2$. Actually, the optimal solution $w_\gamma \in \mathcal{L}_2$ of (12) goes to zero as $\gamma \to \infty$. If it went to some trajectory $w_* \in \mathcal{L}_2$ with $\|w_*\| \ne 0$ since the corresponding output $z_* \in \mathcal{L}_2$ is bounded, the supremum would go to $-\infty$. Then, $\mathcal{H}_2$ performance optimization is readily expressed through (12) as well. We are now in position to state the main result of this note.

Theorem 1. *Let $T > 0$ and $\gamma > 0$ be given. There exist matrices $S_i > 0$ satisfying the two-point boundary value problem (TPBVP) constituted by the coupled differential Riccati equations (10) together with the initial and final boundary conditions*

$$P_i(0) < S_i\ ,\ P_i(T) \ge H_i'S_iH_i, \tag{13}$$

$\forall i \in \mathbb{K}$, if and only if the HMJLS (7)–(9) is mean square stable and the function (12) satisfies

$$\rho_\gamma(\xi_0) < \sum_{i\in\mathbb{K}} \pi_{i0}\xi_0'H_i'S_iH_i\xi_0. \tag{14}$$

Proof. First, define $\nu(t) = (\xi(t), \theta(t), t)$ and

$$V(\nu(t)) = \xi(t)'P_{\theta(t)}(t)\xi(t), \tag{15}$$

$\forall t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$. Due to its time-invariant nature, the solution $P_i(t)$ of the TPBVP is such that $P_i(t) = P_i(t - t_k)$, $\forall i \in \mathbb{K}$ and $\forall t \in [t_k, t_{k+1})$, $\forall k \in \mathbb{N}$ such that $k \ge 1$. As a consequence, $P_i(t_k) = P_i(0)$ and $P_i(t_{k+1}^-) = P_i(T)$, $\forall i \in \mathbb{K}$. Considering (10), due to the fact that (7)–(9) is a particular case of an Itô process, the well known Dynkin's formula, see Costa et al. (2013), implies that, $\forall w \in \mathcal{L}_2$,

$$\begin{aligned} & V(\nu(t_k)) - \mathcal{E}\left\{V(\nu(t_{k+1}^-)) \mid \nu(t_k)\right\} \\ & = \mathcal{E}\left\{\int_{t_k}^{t_{k+1}} \|\gamma^{-1}J_{\theta(t)}'P_{\theta(t)}(t)\xi(t) - \gamma w(t)\|^2 dt \mid \nu(t_k)\right\} \\ & \quad + \mathcal{E}\left\{\int_{t_k}^{t_{k+1}} [z(t)'z(t) - \gamma^2 w(t)'w(t)]dt \mid \nu(t_k)\right\} \\ & \ge \mathcal{E}\left\{\int_{t_k}^{t_{k+1}} [z(t)'z(t) - \gamma^2 w(t)'w(t)]dt \mid \nu(t_k)\right\}. \end{aligned} \tag{16}$$

On the other hand, making use of the initial boundary conditions, for non null trajectories, the quadratic function (15) evaluated at $t = t_k$ is such that $V(\nu(t_k)) < \xi(t_k)'S_{\theta(t_k)}\xi(t_k)$. Analogously, evaluating (15) at t_{k+1}^- implies

$$\begin{aligned} \mathcal{E}\{\ & V(\nu(t_{k+1}^-)) \mid \nu(t_k)\} \\ & = \mathcal{E}\left\{\xi(t_{k+1}^-)'P_{\theta(t_{k+1}^-)}(t_{k+1}^-)\xi(t_{k+1}^-) \mid \nu(t_k)\right\} \\ & \ge \mathcal{E}\left\{\xi(t_{k+1})'S_{\theta(t_{k+1})}\xi(t_{k+1}) \mid \nu(t_k)\right\} \end{aligned} \tag{17}$$

since the final boundary condition from (13) yields $P_{\theta(t_{k+1}^-)}(t_{k+1}^-) \ge H_{\theta(t_{k+1})}'S_{\theta(t_{k+1})}H_{\theta(t_{k+1})}$ and the stochastic process imposes $\theta(t_{k+1}^-) = \theta(t_{k+1})$ with probability one (almost surely). As a consequence, defining $v(\nu(t_k)) \triangleq \xi(t_k)'S_{\theta(t_k)}\xi(t_k)$ and using the lower and upper bounds just calculated, inequality (16) becomes

$$\begin{aligned} & \mathcal{E}\{v(\nu(t_{k+1})) \mid \nu(t_k)\} - v(\nu(t_k)) \\ & < -\mathcal{E}\left\{\int_{t_k}^{t_{k+1}} [z(t)'z(t) - \gamma^2 w(t)'w(t)]dt \mid \nu(t_k)\right\}, \end{aligned} \tag{18}$$

$\forall w \in \mathcal{L}_2$. Because $S_i > 0$, $\forall i \in \mathbb{K}$, then $v(\nu(t_k))$ is positive definite and $\mathcal{E}\{v(\nu(t_k))\}$ can be considered a valid Lyapunov function associated with the discrete-time stochastic process $\xi(t_k) \to \xi(t_{k+1})$, $k \in \mathbb{N}$. Hence, two consequences can be drawn. First, due to the strict inequality in (18), imposing $w \equiv 0 \in \mathcal{L}_2$, there exists $\varepsilon > 0$ sufficiently small such that $\mathcal{E}\{v(\nu(t_{k+1}))|\nu(t_k)\} \le (1 - \varepsilon)v(\nu(t_k))$, which implies that $\mathcal{E}\{v(\nu(t_k))\} \to 0$ as $k \in \mathbb{N}$ goes to infinity. Consequently, $\mathcal{E}\{\|\xi(t)\|^2\} \to 0$ as $t \to \infty$; hence mean square stability holds, see Kushner (1967). Summing up (18) for all $k \in \mathbb{N}$,

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