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Brief paper

Time-optimal velocity tracking control for differential drive robots☆

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ABSTRACT

Nonholonomic wheeled mobile robots are often required to implement control algorithms designed for holonomic kinematic systems. This creates a velocity tracking problem for an actual wheeled mobile robot. In this paper, we investigate the issue of tracking a desired velocity in the least amount of time, for a differential drive nonholonomic wheeled mobile robot with torque inputs. The Pontryagin Maximum Principle provides time-optimal controls that must be implemented as open-loop commands to the motors. We propose two discontinuous state-based feedback control laws, such that the associated closed-loop systems track a desired velocity in minimum time. The feedback control laws are rigorously shown to produce only time-optimal trajectories, by constructing a regular synthesis for each control law. The availability of these time-optimal feedback control laws makes re-computation of open-loop time-optimal controls (due to changes in the desired velocity or input disturbances) unnecessary.

1. Introduction

Differential drive systems are a popular choice for mobile robot platforms. This popularity can largely be attributed to their ability to turn in place, which makes them ideal for navigation in cluttered environments. Another advantage is their simpler mechanical construction, especially when compared to holonomic wheeled mobile robots. The control of nonholonomic wheeled mobile robots has a long history (Bloch, 2003; Brockett, 1983; Park & Kuipers, 2011; Ryan, 1994), with the differential drive robot system being a common example. The most important control problems for this type of robot are the point stabilization problem (Samson, 1991) and the tracking of a reference trajectory (d'Andréa-Novel, Bastin, & Campion, 1992; Fierro & Lewis, 1995; Huang, Wen, Wang, & Jiang, 2014; Jiangdagger & Nijmeijer, 1997).

The field of multi-robot coordination has been an active area of research in recent years. Control methods such as consensus algorithms (Olfati-Saber, Fax, & Murray, 2007) and behavior-based controls (Balch & Arkin, 1998) can achieve a wide variety of tasks. In general, these methods often consider single integrator dynamics, and the commanded control for each robot is a velocity in the plane. Such control laws can be implemented in an exact manner only on holonomic wheeled mobile robots. Furthermore, consider a team of multiple differential drive robots that are to be operated by a human using some input device. Typically, the human may command a motion towards a particular direction. Depending on the headings of the robots, they may or may not be able to move in that direction instantaneously.

In this paper, we are concerned with controlling the planar velocity of the differential drive robot. The goal is to find controls that change the current velocity of the robot to some desired velocity in the plane as fast as possible. The effect of implementing such controls is to make the robots 'appear' to be holonomic, with as small a delay as possible in tracking of commanded velocities. Previous work on time-optimal control for the differential drive robot has focused on control of the robot's position (Reister & Pin, 1994; Renaud & Fourquet, 1997; Van Loock, Pipeleers, & Swevers, 2013), and not its velocity.

In prior work (Poonawala & Spong, 2015), we derived the time-optimal controls for a simplified differential drive robot as functions of time by applying the Pontryagin Maximum Principle. The problem is interesting due to the presence of singular arcs, wherein certain control inputs can be arbitrary functions of time. An important question is related to the robustness of the open-loop controls in Poonawala and Spong (2015) to errors such as switching at the wrong time, input disturbances, and measurement noise. These practical issues may be overcome by defining a (discontinuous) state-based feedback control law such that the resulting closed-loop trajectories from any initial condition are always time-optimal. Such a control law is derived from a *regular synthesis*. Obtaining a regular synthesis from extremal controls

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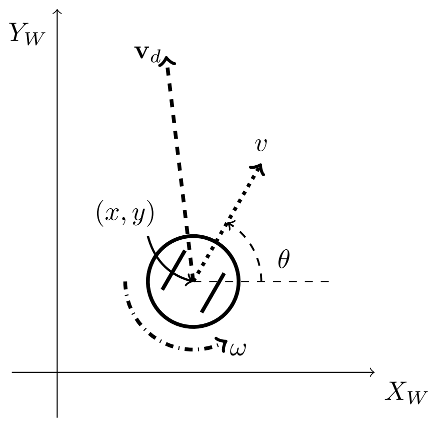

Fig. 1. The differential drive robot with linear speed v, angular velocity ω and desired velocity $\mathbf{v}_d$.

is non-trivial, and is rarely performed in the literature on time-optimal control. Yet, once a regular synthesis has been obtained, a feedback control law which achieves time-optimal transitions can be defined. Furthermore, the synthesis may be used to analyze issues such as robustness to measurement noise.

The main contribution of this paper is to derive a regular synthesis for a torque controlled differential drive system. The regular synthesis is then used to rigorously show that two proposed discontinuous state-based feedback control laws will yield time-optimal velocity tracking for the differential drive robot.

2. Differential drive robot

A sketch of a differential drive robot is shown in Fig. 1. The configuration of the robot is given by $(x, y, \theta) \in \mathbb{R}^3$, where (x, y) is the cartesian position of the centroid of the robot in an inertial reference frame and θ is the heading angle of the robot. Note that the heading angle is measured with respect to the x-axis of the world frame, and depends on the direction of the axis of the wheels. The robot has a forward speed $v \in \mathbb{R}$ and angular velocity $\omega \in \mathbb{R}$ as shown in Fig. 1. The desired velocity $\mathbf{v}_d \in \mathbb{R}^2$ is given by the dashed vector, with magnitude $v_d = \|\mathbf{v}_d\|$. However, the robot's instantaneous velocity lies along the heading direction indicated by the dotted vector, with magnitude v. The magnitude of velocity v and robot heading θ must be controlled such that the robot's instantaneous velocity matches the desired one.

Let u_1 and u_2 be the net torques at the right and left wheels respectively. As described in Sarkar, Yun, and Kumar (1994), the dynamic equations of motion for the differential drive robot are given by

$$m\dot{v} = \frac{r}{2}u_1 + \frac{r}{2}u_2, \tag{1a}$$

$$\dot{\theta} = \omega, \text{ and} \tag{1b}$$

$$J_r\dot{\omega} = \frac{r}{2b}u_1 - \frac{r}{2b}u_2, \tag{1c}$$

where m is the effective mass of the robot, J_r is the effective rotational inertia of the robot about the vertical axis through the center of the wheel base, r is the radius of the wheel, and $2b$ is the distance between the wheels respectively.

2.1. Problem statement

Consider the differential drive robot with dynamics (1). Let the desired velocity be $\mathbf{v}_d$. Let θ_d be the constant angle that $\mathbf{v}_d$ makes with the x-axis of the coordinate system in which (x, y) is defined. Let the torque inputs u_1 and u_2 satisfy $\|u_1\| \le u_m$ and $\|u_2\| \le u_m$ respectively, where $u_m > 0$ denotes the maximum achievable torque at each wheel. Derive a state-based feedback control law such that the $\theta(t) \to \theta_d$ and $v(t) \to \|\mathbf{v}_d\|$ in the least amount of time possible.

3. Extremal controls

The state of the differential drive robot is taken as $q = (v, \theta, \omega)^T \in \mathbb{R}^3$, and its dynamics $\dot{q} = f(q, u)$ are given by (1c). Note that θ is treated as a real number instead of an element of the manifold $\mathcal{S}^1$. The input space $U \subset \mathbb{R}^2$ is $[-u_m, u_m] \times [-u_m, u_m]$. The problem in Section 2.1 is equivalent to finding a control input that results in a trajectory which begins in initial state $q_0 = (v_0, \theta_0, \omega_0) \in \mathbb{R}^3$ and reaches a desired final state $q_d = (\|\mathbf{v}_d\|, \theta_d, 0) \in \mathbb{R}^3$ in the least amount of time possible. We will say that such a control achieves a *transition* from q_0 to q_d, in minimum time.

Since $f(q, u) = f(q + [v, \theta, 0]^T, u)$, we can change the origin of the coordinate system such that the desired final state becomes the origin $(0, 0, 0)$. This change of origin does not change the time-optimal control associated with the desired transition between states. The problem in Section 2.1 then becomes that of finding a time-optimal control that results in a transition to the origin for any initial state in $\mathbb{R}^3$. For the remainder of the paper we denote the new initial state as q_0, and the new final state as q_d, which is now the origin.

An *admissible control* $u(t)$ defined on any finite time interval $I = [0, T]$ is one such that $u(t) \in U$ for all $t \in I$. Let $u(t)$ defined on I be an admissible control which achieves such a transition from q_0 to the origin. The pair $(q(t), u(t))$ is referred to as a *controlled trajectory* from q_0 to the origin. The final time T for a transition from q_0 to the origin depends on the control and q_0, but we suppress this dependence when using the symbol I.

The Pontryagin Maximum Principle (Mauder, 2012; Schattler & Ledzewicz, 2012; Sussmann & Tang, 1991; Wu, Chen, & Woo, 2000) can be used to find the time-optimal controlled trajectories, since it specifies necessary conditions that the trajectories must satisfy. Any controlled trajectory meeting the necessary conditions is called an *extremal*. In the case of extremal $(q^*(t), u^*(t))$, we refer to $q^*(t)$ as the *extremal trajectory* and $u^*(t)$ as the *extremal control*.

The application of the Pontryagin Maximum Principle results in the conclusion that all extremal controls defined over time interval I consist of only two possible types (Poonawala & Spong, 2015):

- $C1$ At least one motor has a constant torque with value u_m or $-u_m$ over I.
- $C2$ Both motors have piecewise constant torques (with possible values in $\{-u_m, +u_m\}$) with exactly one switch for each motor at time instants t_1 and t_2 such that $t_1, t_2 \in (0, T)$.

The $C1$ control corresponds to a *singular* extremal control, since although one motor torque is constant, the other motor torque can be an arbitrary function of time. It is the presence of singular extremals that makes this problem interesting. As shown in Poonawala and Spong (2015), for every transition corresponding to a $C1$ control where one motor torque is arbitrary, there is a $C1$ control that achieves the same transition in the same time, where that motor switches no more than twice between its maximum values. Therefore, we can now only consider extremals that consist of no more than two time-instants of switching. We can represent an extremal as a sequence of time-intervals called *phases*. During each phase, the control input is constant. The switching of the motor torques occurs when the phase changes. There will be no more than three phases for any such extremal.