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Brief paper

Impulsive stabilization of a class of singular systems with time-delays☆

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ABSTRACT

This paper deals with the impulsive stabilization problem for a class of linear singular systems with time-delays. The stabilization is achieved by only exerting impulsive action on the slow state variables. Two novel Lyapunov methods are presented to determine exponential stability of the impulsively controlled systems. For the case where the time-delay is unknown and may be time-varying, a Lyapunov–Razumikhin method is developed, in which the Razumikhin condition is constructed by exploiting the relation among the fast state variables, the slow state variables, and their initial values. For the case where the delay derivative is strictly less than 1, a descriptor type of impulse-time-dependent Lyapunov functional is introduced, which is discontinuous at impulse times but does not grow along the state trajectories by construction. By using a convex technique, the stability criteria are expressed in terms of linear matrix inequalities (LMIs). Then, the impulsive controllers can be designed in the framework of LMIs. The effectiveness and advantages of the proposed methods are confirmed through simulation results.

1. Introduction

During the last decades, singular systems have been widely investigated due to their important applications in many fields such as economic systems, power systems, electrical networks, mechanical systems, chemical engineering systems, etc. Singular systems are also called descriptor systems, implicit systems, generalized state-space systems, or differential–algebraic systems, according to the area of applications. The advantage of singular systems is that they are capable to preserve the structure of physical systems and to describe both dynamic and static constraints. A variety of works on stability analysis and synthesis of singular systems have been reported in the literature (see (Chadli & Darouach, 2014; Dai, 1989; Liberzon & Trenn, 2012; Masubuchi, Kamitane, Ohara, & Suda, 1997; Mironchenko, Wirth, & Wulff, 2015; Wu, Shi, & Gao, 2010; Wu, Su, & Shi, 2012; Wu & Zheng, 2009; Zhou, Ho, & Zhai, 2013), and the references therein). In practical applications, time delays are frequently encountered, which typically arise due to information transmission among different parts of the system. As pointed out in Fridman (2002), small delay in the feedback may destabilize the trivial solution of singular systems. For this reason, analysis and control design for time-delay singular systems have become an important topic in control theory. In recent years, interesting results on stability and control problems have been developed for linear time-delay singular systems in the linear matrix inequalities (LMIs) framework (Du, Yue, & Hu, 2014; Fridman & Shaked, 2002; Wu, Su, & Chu, 2010; Xu, Van Dooren, Stefan, & Lam, 2002; Zhu, Zhang, Cheng, & Feng, 2007).

On the other hand, considerable attention has been paid to impulsive control of linear and nonlinear systems. Generally speaking, impulsive control is to externally exert control action on the state trajectories at some discrete instants, according to given specifications. This presents an important advantage as it provides an effective way to deal with the systems which cannot endure continuous inputs. So impulsive control has been commonly applied to various fields such as mechanical systems, communication security systems, orbit control, etc. Several efficient methods have been developed for stability analysis of the impulsively controlled (regular) systems with/without delays: the Razumikhin-type Lyapunov function method (Chen & Zheng, 2009, 2011; Liu & Ballinger, 2001), the impulsive delay differential inequality method (Li & Song, 2017; Yang & Xu, 2007), the looped-functional method (Briat & Seuret, 2012a,b), the impulse-time-dependent Lyapunov function/functional method (Chen, Li, & Lu, 2013), and

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the polytopic embedding method (Hetel, Daafouz, Tarbouriech, & Prieur, 2013). Despite an increased interest in impulsive control problems, there are only a few results available for impulsive stabilization of singular systems. In Shi, Zhang, Yuan, and Liu (2011), an impulsive feedback element was introduced in the hybrid control law to eliminate the state jumps for switched singular systems. An impulsive control strategy was proposed in Feng and Cao (2015) to stabilize a class of nonlinear switched singular systems. However, these works do not consider the effects of time delays.

Different from the undelayed singular systems, the fast state variables of time-delay singular systems are governed by a continuous difference equation. Since the dynamical behavior of continuous difference equations cannot be controlled by discrete impulses, the pure impulsive control method is suitable only to the time-delay singular systems in which the continuous difference system have some sort of stability property. When exerting the impulsive control on the fast state variables, the resulting controlled system is an interconnection of an impulsive time-delay system and a continuous difference system. A major difficulty for stability analysis of the impulsively controlled time-delay singular systems lies in the fact that the dynamical behavior of the fast state variables is characterized by continuous difference equations rather than differential equations. When there is no impulsive effect, one way to overcome such difficulty is to employ a descriptor-type Lyapunov–Krasovskii functional for estimating the slow state variables. Then Barbalat's lemma is used to deduce asymptotic convergence of the fast state variables (Fridman & Shaked, 2002; Xu et al., 2002). Because this method requires the delay derivatives not to be big, it is not applicable to singular systems with fast-varying delays (without any constraint on the delay derivative). A widely used method for stability analysis of regular systems with fast-varying delays is to apply the Razumikhin technique for treating the delayed terms. The key to the Razumikhin technique is to ensure that the delayed terms are dominated by the undelayed terms. It is worth mentioning that the descriptor-type Lyapunov functions do not include the information of the fast state variables. So when applying the Razumikhin technique to time-delay singular systems, the delayed terms of fast state variables cannot be dominated by the undelayed terms via the descriptor-type Lyapunov functions. This means that the standard Razumikhin technique is no longer workable for time-delay singular systems with fast-varying delays. Therefore, the impulsive stabilization problem for time-delay singular systems is not trivial, and still remains a technically challenging issue.

Given the above analysis, the focal point of this paper is on developing novel Lyapunov methods to establish impulsive stabilization conditions of linear time-delay singular systems. In the case of unknown and time-varying time-delay, by building up the relation between the fast state variables and the slow state variables through a continuous difference inequality, a Lyapunov–Razumikhin method is proposed to analyze the stability of the impulsively controlled singular systems. In the case of the delay derivative being strictly less than one, a descriptor-type impulse-time-dependent Lyapunov functional based method is developed to derive a new condition for impulsive stabilization. The novelty of the proposed methods is their ability to capture the hybrid structure characteristics of the impulsively controlled singular systems. In relation to impulsive stabilization, design of state feedback impulsive controllers is also studied. The impulsive gain matrices can be obtained by solving a set of LMIs.

The rest of the paper is organized as follows. In the next section, the model of impulsively controlled time-delay singular systems and some preliminaries are presented. In Section 2, stability criteria for the considered systems are developed by using the Razumikhin–Lyapunov function and Lyapunov functional methods. Section 4 is devoted to the design of state-feedback impulsive controllers. In Section 5, two numerical examples are given to demonstrate the efficiency of the proposed methods. Finally, some concluding remarks are made in Section 6.

2. System description and preliminaries

In the sequel, if not explicitly, matrices are assumed to have compatible dimensions. The notation $M > (\geq, <, \leq) 0$ is used to denote a symmetric positive-definite (positive-semidefinite, negative, negative-semidefinite) matrix. I stands for an identity matrix of suitable dimension. $\|\cdot\|$ refers to the Euclidean vector norm. $\mathbb{N}$ represents the set of positive integers. For $\tau > 0$, let $C([-\tau, 0], \mathbb{R}^n)$ denote the space of bounded, continuous functions $x : [-\tau, 0] \mapsto \mathbb{R}^n$ with norm $\|x\|_\tau = \max_{-\tau \leq \theta \leq 0} \|x(t+\theta)\|$. If $y \in C([-\tau, \alpha], \mathbb{R}^n)$ with $\alpha > 0$ and $t \in [0, \alpha)$, then $y_t \in C([-\tau, 0], \mathbb{R}^n)$ is defined by $y_t(\theta) = y(t+\theta),\ \theta \in [-\tau, 0]$.

Consider an impulsively controlled linear time-delay singular system with the form of

$$\begin{aligned} E\dot{x}(t) &= A_0 x(t) + A_1 x(t-\tau(t)) \\ &\quad + EB\sum_{k=1}^{\infty} u(t)\delta(t-t_k),\ t > 0, \\ x(t) &= \phi(t) = \mathrm{col}(\phi_1(t), \phi_2(t)),\ \ -\bar{\tau} \leq t \leq 0, \end{aligned} \tag{1}$$

where $x(t) = \mathrm{col}(x_1(t), x_2(t)) \in \mathbb{R}^n$ is the state, in which $x_i(t) \in \mathbb{R}^{n_i}$, $i = 1, 2$, and $n_1 + n_2 = n$; $u(t) \in \mathbb{R}^m$ is the impulsive control input. The singular matrix E and the matrices A_0, A_1, B are constant matrices with appropriate dimensions. The sequence of $\{t_k\}$ denotes the impulse instants satisfying $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$ and $\lim_{k\to\infty} t_k = \infty$. The time-varying function $\tau(t)$ is the state delay satisfying $\tau_0 \leq \tau(t) \leq \bar{\tau}$, where τ_0 and $\bar{\tau}$ are positive scalars, and $\phi_i \in C([-\bar{\tau}, 0]; \mathbb{R}^{n_i})$, $i = 1, 2$, are the initial functions. Without loss of generality, assume that the singular matrix E takes the form of $E = \mathrm{diag}\left(I_{n_1}, 0_{n_2}\right)$, and the matrices in (1) have the following structure:

$$A_i = \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix},\ i = 0, 1,\ B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}.$$

Moreover, we adopt the following assumption.

Assumption (A1): A_{04} is nonsingular.

This paper is concerned with designing a linear reduced-order state-feedback impulsive controller

$$u(t_k) = [K \quad 0]x(t_k^-),\ k \in \mathbb{N}, \tag{2}$$

where $K \in \mathbb{R}^{m\times n_1}$, and $x(t^-) = \lim_{h\to 0^+} x(t-h)$. The connection of system (1) and controller (2) gives

$$\begin{aligned} E\dot{x}(t) &= A_0 x(t) + A_1 x(t-\tau(t)),\ t \neq t_k, \\ \Delta x_1(t) &= B_1 K x_1(t^-),\ t = t_k,\ k \in \mathbb{N}, \\ x(t) &= \phi(t),\ \ -\bar{\tau} \leq t \leq 0, \end{aligned} \tag{3}$$

where $\Delta x_1(t) = x_1(t^+) - x_1(t^-)$ with $x_1(t^+) = \lim_{h\to 0^+} x_1(t+h)$. Throughout the paper, we assume that $x(t) = x(t^+)$, i.e., the solutions of (3) are right continuous. Define the compatible initial function space $\mathcal{C}_E = \{\phi \in PC[-\bar{\tau}, 0] : A_{03}\phi_1(0) + A_{04}\phi_2(0) + A_{13}\phi_1(-\tau(0)) + A_{14}\phi_2(-\tau(0)) = 0\}$. Set

$$\bar{A}_0 = -A_{04}^{-1}A_{03},\ \bar{A}_1 = -A_{04}^{-1}A_{13},\ \bar{A}_2 = -A_{04}^{-1}A_{14}.$$

Lemma 1. *Consider system (3) satisfying assumption* (**A1**). *Then for any $\phi \in \mathcal{C}_E$, the solution to (3) exists and is unique on $\mathbb{R}_+$.*

Proof. For $t \in [0, \tau_0]$, Eq. (3) is equivalent to

$$\begin{aligned} \dot{x}_1(t) &= \sum_{i=1}^{2} A_{0i}x_i(t) + \sum_{i=1}^{2} A_{1i}\phi_i(t-\tau(t)),\ t \neq t_k, \\ 0 &= \sum_{i=3}^{4} A_{0i}x_{i-2}(t) + \sum_{i=3}^{4} A_{1i}\phi_{i-2}(t-\tau(t)), \\ x_1(t) &= (I + B_1K)x_1(t^-),\ t = t_k. \end{aligned} \tag{4}$$