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Brief paper

Smooth control design for adaptive leader-following consensus control of a class of high-order nonlinear systems with time-varying reference☆

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ABSTRACT

In this paper, we investigate the distributed adaptive leader-following consensus control for high-order nonlinear multi-agent systems with time-varying reference trajectory under directed topology subjected to mismatched unknown parameters and uncertain external disturbances. By introducing local estimators for the bounds of reference trajectory and a filter for each agent, a new backstepping based smooth distributed adaptive control protocol is proposed. It is shown that global uniform boundedness of all the closed-loop signals and asymptotically output consensus tracking can be achieved. Simulation results are provided to verify the effectiveness of our scheme.

1. Introduction

Consensus of multi-agent systems, due to its wide potential applications, has become a rapidly emerging topic in various research communities over the past decades. Distributed consensus control normally aims at achieving an agreement for the states or the outputs of network connected systems, by designing controller for each agent based on only locally available information collected within its neighboring area. This control issue can be further classified into leaderless consensus control (e.g. Ren and Beard, 2005 and many other references) and leader-following consensus control, such as Arcak (2007), Hong, Hu, and Gao (2006), Huang, Wen, Wang, and Song (2015), Wang, Huang, Wen, and Fan (2014), Wang, Wen, and Huang (2016), Wang, Wen, Huang, and Li (2016), Yoo (2013), Zhang, Feng, Yang, and Liang (2015), Zhang, Jiang, Luo, and Xiao (2016), Zhang and Lewis (2012), Zhang, Liu, and Feng (2015), Zhang, Zhang, Yang, and Luo (2015), Wang, Song, and Lewis (2015), Wang, Song, Krstic, and Wen (2016a), Wang, Song, Krstic, and Wen (2016b), Wang and Song (2017) and Wang et al. (2017). Note that in most of currently available results on the latter issue, the desired references are set by the behaviors of specific leaders with similar dynamics to the followers and zero/known inputs.

Several researchers have considered more general cases of leader-following consensus control with time-varying trajectories, i.e., $y_r(t)$, which is only available to part of followers. For example, in Li, Liu, Ren, and Xie (2013), the distributed tracking control problem of multi-agent systems is considered with general linear dynamics and a leader whose control input is nonzero and not available to any follower. However, the control signals of the agents are non-smooth. Similarly in Lu, Chen, and Chen (2016) two non-smooth leader-following formation protocols for nonidentical Lipschitz nonlinear multi-agent systems are presented with directed communication network topologies. In Wang et al. (2014) and Yu and Xia (2012), the reference trajectory is linearly-parameterized by some basic functions which are known to all agents and a distributed adaptive control approach based on backstepping technique is proposed. In Yoo (2013) the distributed consensus tracking control problem for multiple strict-feedback systems with unknown nonlinearities under a directed graph topology is studied by adopting dynamic surface design approach and semi-globally uniformly ultimately bounded consensus tracking errors are finally obtained.

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However, the available results on the leader-following consensus with time-varying trajectory under directed topology are still unsatisfactory, for example, the obtained control signals are non-smooth, e.g. Li & Liu et al. (2013) and Lu et al. (2016), or consensus errors could not be asymptotically stable, e.g. Yoo (2013) and Yu and Xia (2012). To address these issues, a new distributed adaptive backstepping based control scheme is developed in this paper to achieve asymptotical consensus for nonlinear multi-agent systems under directed topology while guaranteeing that the protocols are smooth. The main contributions of this paper can be summarized as follows.

• The non-smooth signum function based distributed control approaches in Dong (2012), Li and Liu et al. (2013), Lu et al. (2016) and Mei, Ren, and Ma (2011) are undesired due to chattering phenomenon. To address this issue, new compensating terms are introduced including some smooth functions of consensus errors and a positive integrable time-varying signal, with which smooth consensus controllers are obtained. Furthermore, different from El-Ferik, Qureshi, and Lewis (2014), Yoo (2013) and Zhang and Lewis (2012), global uniform boundedness of all the closed-loop signals and asymptotically consensus tracking for all agent outputs are achieved in this paper.

• In contrast to Bai, Arcak, and Wen (2009), Hu and Zheng (2014), Wang et al. (2014) and Yu and Xia (2012), the assumptions of linearly parameterized reference signals and the corresponding basis function vectors being known by all agents are no longer needed. In this paper, it is assumed that the desired trajectory $y_r(t)$ is known exactly for only part of the agents in the group.

• The considered multi-agent system model is more general than those in most existing results on distributed consensus control including Arcak (2007), Bai et al. (2009), Das and Lewis (2010), Hong et al. (2006) and Ren (2007) in the following terms. (i) The agents are nonlinear and allowed to have arbitrary relative degree and nonidentical dynamics; (ii) Intrinsic mismatched unknown parameters and uncertain disturbances are simultaneously involved.

Finally, simulation results on an application example are provided to verify the effectiveness of the proposed distributed adaptive control scheme.

2. Problem formulation

2.1. System model

We consider a group of N nonlinear agents which can be modeled as follows.

$$\begin{aligned}\dot{x}_{i,q} &= x_{i,q+1} + \varphi_{i,q}(x_{i,1},\ldots,x_{i,q})^T\theta_i, \quad q=1,\ldots,n-1\\ \dot{x}_{i,n} &= u_i + \varphi_{i,n}(x_i)^T\theta_i + d_i(t)\\ y_i &= x_{i,1}, \quad \text{for } i=1,2,\ldots,N\end{aligned} \tag{1}$$

where $x_i = [x_{i,1},\ldots,x_{i,n}]^T \in \Re^n$, $u_i \in \Re$, $y_i \in \Re$ are the state, control input and output of the ith agent, respectively. $\theta_i \in \Re^{p_i}$ is a vector of unknown constants. $\varphi_{i,j} : \Re^j \to \Re^{p_i}$ for $j = 1,\ldots,n$ are known smooth nonlinear functions. $d_i(t)$ represents the external disturbance.

2.2. Information transmission among the N agents

Suppose that the communications among the N agents can be represented by a directed graph $\mathcal{G} \triangleq (\mathcal{V},\mathcal{E})$ where $\mathcal{V} = \{1,\ldots,N\}$ denotes the set of indexes (or vertices) corresponding to each agent, $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ is the set of edges between two distinct agents. An edge $(i,j) \in \mathcal{E}$ indicates that agent j can obtain information from agent i, but not necessarily vice versa (Ren and Cao, 2010). In this case, agent i is called a neighbor of agent j. We denote the set of neighbors for agent i as $\mathcal{N}_i$. Self edges (i,i) are not allowed in this paper, thus $(i,i) \notin \mathcal{E}$ and $i \notin \mathcal{N}_i$. The connectivity matrix $A = [a_{ij}] \in \Re^{N\times N}$ is defined such that $a_{ij} = 1$ if $(j,i) \in \mathcal{E}$ and $a_{ij} = 0$ if $(j,i) \notin \mathcal{E}$. Clearly, the diagonal elements $a_{ii} = 0$. We introduce an in-degree matrix $\triangle$ such that $\triangle = \mathrm{diag}(\triangle_i) \in \Re^{N\times N}$ with $\triangle_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ being the ith row sum of A. Then, the Laplacian matrix of $\mathcal{G}$ is defined as $\mathcal{L} = \triangle - A$. A direct path from agent i to agent j is a sequence of successive edges in the form $\{(v_i,v_l),(v_l,v_m),\ldots,(v_k,v_j)\}$. A digraph has a spanning tree, if there is an agent called root, such that there is a directed path from the root to each other agent in the graph.

We now use $\mu_i = 1$ to indicate the case that y_r is accessible directly to agent i; otherwise, μ_i is set as $\mu_i = 0$. Throughout this paper, the following notations are used. $\|\cdot\|$ is the Euclidean norm of a vector. Let $a \in \Re^n$ and $b \in \Re^n$ being two vectors, then define the vector operator $.*$ as $a.*b = [a(1)b(1),\ldots,a(n)b(n)]^T$. Let Q being a matrix, then $\lambda_{\min}(Q)$ denotes the minimum eigenvalue of Q.

The *control objective* is, while only part of the followers have access to the leader, to design distributed adaptive smooth controllers u_i for each agent by utilizing only locally available information obtained from the intrinsic agent and its neighbors such that:
(i) all the signals in the closed-loop system are globally uniformly bounded;
(ii) the outputs of all the overall systems can track the desired trajectory $y_r(t)$ asymptotically, i.e. $\lim_{t\to\infty}[y_i(t) - y_r(t)] = 0,\ \forall i$.

To achieve the objective, the following assumptions are imposed.

Assumption 1. The first nth-order derivatives of $y_r(t)$ are bounded, piecewise continuous. Let $F_j, j = 1,\ldots,n$ being the bound of jth-order derivative of $y_r(t)$, then F_j is also available to agent i if $\mu_i = 1$.

Assumption 2. The directed graph $\mathcal{G}$ contains a spanning tree with the root agent being the leader.

Remark 1. As pointed in Section 1, the multi-agent system model described in (1) is more general than those in most of the currently available results on distributed consensus control including (Arcak, 2007; Bai et al., 2009; Das & Lewis, 2010; Hong et al., 2006; Yu & Xia, 2012) for the following terms. (i) The subsystems are nonlinear and allowed to have arbitrary relative degree and nonidentical dynamics; (ii) intrinsic mismatched unknown parameters and uncertain disturbances are simultaneously involved.

Remark 2. Assumption 1 indicates that the bounds of up to nth-order derivatives of $y_r(t)$ are available to agent i if $\mu_i = 1$. This is a mild assumption because it is a common case that part of the agents may have full knowledge of the reference trajectory $y_r(t)$.

The following lemma brought from Zhang and Lewis (2012) is then introduced, which will be useful in our design and analysis of the distributed adaptive controllers.

Lemma 1. *Based on Assumption 2, the matrix $(\mathcal{L}+\mathcal{B})$ is nonsingular where $\mathcal{B} = diag\{\mu_1,\ldots,\mu_N\}$. Define*

$$\begin{aligned}\bar{q} &= [\bar{q}_1,\ldots,\bar{q}_N]^T = (\mathcal{L}+\mathcal{B})^{-1}[1,\ldots,1]^T\\ P &= diag\{P_1,\ldots,P_N\} = diag\left\{\frac{1}{\bar{q}_1},\ldots,\frac{1}{\bar{q}_N}\right\}\\ Q &= P(\mathcal{L}+\mathcal{B}) + (\mathcal{L}+\mathcal{B})^T P,\end{aligned} \tag{2}$$

then $\bar{q}_i > 0$ for $i = 1,\ldots,N$ and Q is positive definite.