ARTICLE IN PRESS

Automatica ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Technical communique

Input-constrained multi-model unfalsified switching control[☆]

Mojtaba Nouri Manzar[a], Giorgio Battistelli[b], Ali Khaki Sedigh[a]

[a] K.N. Toosi University of Technology, Seyed-Khandan Bridge, Shariati Ave., Tehran, Iran

[b] University of Florence, Via di Santa Marta 3, 50139, Firenze, Italy

ARTICLE INFO

Article history:
Received 3 March 2016
Received in revised form
18 March 2017
Accepted 4 April 2017
Available online xxxx

Keywords:
Unfalsified control
Multi-model unfalsified control
Input constraint
Switching supervisory control

ABSTRACT

This note deals with the problem of controlling an uncertain multivariable plant in the presence of input saturation via switching among a finite family of controllers having a generalized anti-windup architecture. The problem is addressed within the multi-model unfalsified adaptive switching control framework. It is shown that proper definitions of fictitious references and test functionals allow to prove stability of the overall switching scheme, provided that at least one controller in the finite family is stabilizing. The satisfiability of this assumption is discussed and simulation results are reported.

1. Introduction

Unfalsified adaptive switching control (UASC) is a multi-controller supervisory control technique that can handle wide range uncertainties by resorting to the concept of controller falsification (Battistelli, Mosca, Safonov, & Tesi, 2010; Wang, Paul, Stefanovic, & Safonov, 2007). In UASC, the computation of certain fictitious reference signals makes it possible to evaluate in real-time the potential performance of a given candidate controller by using only plant input/output data. While originally UASC is a model-free approach, in Baldi, Battistelli, Mosca, and Tesi (2010); Battistelli, Mosca, and Tesi (2014) it is shown how to safely introduce a finite family of models within the UASC framework. The resulting multi-model UASC (MMUASC) scheme enjoys the positive features of both UASC and multi-model ASC (Morse, Mayne, & Goodwin, 1992), i.e., it ensures stability irrespective of the model distribution and shows improved performance when the distance between the true plant and the model family is small.

This note discusses how MMUASC can be extended so as to handle the presence of input constraints. In fact, physical constraints are unavoidable for implementing a control strategy and, in order to preserve stability, such constraints should be taken into account when designing and analyzing a control scheme (see Liu, Chitour, & Sontag, 1996; Saberi, Stoorvogel, & Sannuti, 2012, and the references therein). In particular, we focus on a MMUASC scheme equipped with an anti-windup structure (Kothare, Campo, Morari, & Nett, 1994), which is a common approach for input constraint compensation. Further, we suppose that the information on the uncertain plant allows one to design a set of candidate controllers with the property that in all the possible operating conditions there always exists at least one stabilizing controller (problem feasibility). Since we deal with global stability, this requires the uncertain plant to be ANCBC (asymptotic null controllable with bounded input) (Saberi et al., 2012). We show that the use of an anti-windup structure calls for special care in the definition of the fictitious reference signals, since the controllers are not invertible, and of the test functionals. The proposed choices allow us to prove stability under the minimal assumption of problem feasibility. In this respect, we also show that a finite controller family ensuring problem feasibility always exists when the uncertain plant is neutrally stable and the uncertainty set is compact.

We note that, in the context of fault-tolerant control, a lot of attention has been devoted to the development of adaptive control techniques (including multi-model approaches) able to deal with actuator limitations, typically unknown and unpredictable loss of effectiveness of some actuators (Boskovic & Mehra, 2002; Tang, Tao, & Joshi, 2007). However, the setting considered in this paper is different in that we consider a limitation of the saturation type affecting all the actuators and adaptation is used to face the uncertainty in the plant transfer function. In this context, the derived stability result represents a significant improvement, since the

[☆] The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Tingshu Hu under the direction of Editor André L. Tits.

E-mail addresses: nourimanzar@gmail.com (M. Nouri Manzar), giorgio.battistelli@unifi.it (G. Battistelli), sedigh@kntu.ac.ir (A. Khaki Sedigh).

http://dx.doi.org/10.1016/j.automatica.2017.04.044
0005-1098/

stability properties of existing approaches dealing with a similar setting (De Persis, De Santis, & Morse, 2004; Kim, Yoon, & De Persis, 2007; Kim, Yoon, Shim, & Seo, 2008) are guaranteed only under exact model-matching (i.e., when the uncertain plant is neutrally stable and coincides with one of the considered nominal models).

2. Problem statement

The plant P to be controlled consists of the cascade of an unknown LTI system with an input non-linearity

$$P : \begin{cases} x(t+1) = Ax(t) + B\varphi(u(t)) \\ y(t) = Gx(t) \end{cases} \tag{1}$$

where $u \in \mathbb{R}^m$ is the input, $y \in \mathbb{R}^p$ the output, and $x \in \mathbb{R}^n$ the state. The uncertain matrices (A, B, G) belong to an uncertainty set $\mathscr{S}$. The input non-linearity $\varphi : \mathbb{R}^m \to \mathbb{R}^m$ is supposed to be of saturation type in the sense that $\varphi(u) = u$ for any $u \in \mathbb{U}$, and $\varphi(u) \in \mathbb{U}$ for any $u \in \mathbb{R}^m$ for some compact set $\mathbb{U}$, with 0 belonging to the interior of $\mathbb{U}$. Hereafter, for ease of presentation, $\mathbb{U}$ is taken as a hyper-rectangle $\mathbb{U} = [\underline{u}^{(1)}, \bar{u}^{(1)}] \times \cdots \times [\underline{u}^{(m)}, \bar{u}^{(m)}]$.

The switching controller C_σ is supposed to have a generalized anti-windup structure (see Kothare et al., 1994). Specifically, let a finite family of two-degrees-of-freedom LTI controllers $\mathscr{C} = \{C_1, \dots, C_N\}$ be available. Further, for each controller C_i, let the transfer function $K_i(d)$ from (y, w), with w the reference input, to the unsaturated control input v be represented according the coprime Matrix Fraction Description (MFD) $K_i(d) = R_i^{-1}(d)[S_i(d)\ T_i(d)]$ where d is the unit backward shift operator. The transfer functions $R_i(d)$, $S_i(d)$, and $T_i(d)$ are stable and $R_i(d)$ monic, i.e., $R_i(0) = I$. The switching controller C_σ is realized as

$$\begin{aligned} u(t) &= \hat{\varphi}(v(t)) \\ v(t) &= (I - R_{\sigma(t)}(d))u(t) + T_{\sigma(t)}(d)w(t) + S_{\sigma(t)}(d)y(t) \end{aligned} \tag{2}$$

where $\sigma : \mathbb{Z}^+ \to \{1, \dots, N\}$ is the controller switching signal. The subscript $\sigma(t)$ identifies the candidate controller $C_{\sigma(t)}$ connected to the plant at time t, and Eq. (2) is intended as a shorthand notation to mean that, over each interval of time where $\sigma(t) = i$ is constant, $v(t)$ is the output of a LTI system satisfying the difference equation $v(t) = (I - R_i(d))u(t) + T_i(d)w(t) + S_i(d)y(t)$ with the state at the beginning of the interval initialized according to some rule (e.g., via a shared-state architecture).

The function $\hat{\varphi} : \mathbb{R}^m \to \mathbb{R}^m$ is an artificial non-linearity that preserves direction of input as proposed in Campo and Morari (1990): $\hat{\varphi}(v)$ takes value v if $v \in \hat{\mathbb{U}}$ and value $v \min_j\{\varphi(v^{(j)})/v^{(j)}\}$ otherwise, where $v^{(j)}$ denotes the jth component of v. The compact set $\hat{\mathbb{U}}$ limiting the control signal of the artificial non-linearity is supposed to satisfy the condition $\hat{\mathbb{U}} \subseteq \mathbb{U}$ so that the function φ always remains in linear zone, i.e., $\varphi(\hat{\varphi}(v(t))) = \hat{\varphi}(v(t))$.

In the following, the time-varying feedback system consisting of the feedback interconnection of the unknown plant (1) and the switching controller (2) will be denoted by $(P/C_{\sigma(t)})$. In order to decide which controller should be active at each instant, a high-level unit, called supervisor, computes in real-time data-driven test functionals $J_i(t)$, each one related to one of the candidate controller C_i. The supervisor compares the test functionals and selects the controller index $\sigma(t)$ via the hysteresis switching logic (HSL) (Morse et al., 1992)

$$\sigma(t+1) = \arg\min_i (J_i(t) - h\delta_{i,\sigma(t)}) \tag{3}$$

where $\delta_{i,j}$ is Kronecker delta and h the hysteresis constant. Then the problem we address is how to select the test functionals so as to ensure that $(P/C_{\sigma(t)})$ is stable.

3. Test functionals and stability analysis

Let $\mathscr{M} = \{M_i, i = 1, \dots, N\}$ be a finite family of plant models of the form (1) with matrices (A_i, B_i, G_i), and let each controller C_i be designed so that the closed loop (M_i/C_i) is stable and has satisfactory performance. In MMUASC, each test functional J_i is defined in terms of the discrepancy between the *reference loop* (M_i/C_i) and the *potential loop* (P/C_i). In this way, the supervisor can select a controller such that (P/C_σ) behaves as close as possible to one of the reference loops.

The fundamental idea of UASC is to exploit the information on the unknown plant P contained in the recorded I/O sequence (u, y) in order to evaluate the behavior of the potential loop (P/C_i) in response to a suitable *fictitious reference* w_i. In fact, when w_i is chosen so as to satisfy the relationship

$$\begin{aligned} u(t) &= \hat{\varphi}(T_i(d)w_i(t) + f_i(t)) \\ f_i(t) &= S_i(d)y(t) + (I - R_i(d))u(t), \end{aligned} \tag{4}$$

by construction it turns out that the hypothetical response of the potential loop (P/C_i) to the fictitious reference w_i would coincide precisely with the recorded I/O sequence (u, y). In other words, the collected data allows one to infer how (P/C_i) would respond to the signal w_i.

In this respect, it can be readily seen that the computation of w_i gives rise to two problems. The first one is that the transfer function $T_i(d)$ need not be stably invertible, while the second one is that the non-linear function $\hat{\varphi}(\cdot)$ is clearly non-invertible so that the fictitious reference is not unique. The first problem can be easily circumvented by computing the signal $\beta_i(t) = T_i(d)w_i(t)$ instead of $w_i(t)$, thus avoiding the necessity of inverting $T_i(d)$. As for the second problem, clearly when $u(t)$ belongs to $\hat{\mathbb{U}}^\circ$, i.e., the interior of $\hat{\mathbb{U}}$, the solution is unique and given by

$$\beta_i(t) = u(t) - f_i(t). \tag{5}$$

On the other hand, when $u(t)$ belongs to $\partial\hat{\mathbb{U}}$, i.e., the boundary of $\hat{\mathbb{U}}$, many solutions are possible. A reasonable choice amounts to selecting, among all the signals $\beta_i(t)$ satisfying (4), the one which is closest to the true signal $\hat{\beta}_i(t) = T_i(d)\,w(t)$ which would be generated in case controller C_i were active. In practice, we solve the optimization problem

$$\begin{aligned} &\arg\min_{\beta_i(t)} |\beta_i(t) - \hat{\beta}_i(t)|^2 \\ &\text{s.t. } \beta_i(t) = \alpha u(t) - f_i(t), \quad \alpha \geq 1 \end{aligned} \tag{6}$$

where $|\cdot|$ is Euclidean norm and the constraint is imposed so as to preserve the input direction. The optimization admits the analytic solution

$$\beta_i^{opt}(t) = \max\left\{1, \frac{\sum_j u^{(j)}(\hat{\beta}_i^{(j)} + f_i^{(j)})}{\sum_j (u^{(j)})^2}\right\} u(t) - f_i(t) \tag{7}$$

where the superscript (j) denotes the jth component. Notice that the signal $\beta_i(t)$ is used only in the computation of the test functionals, as detailed below, and hence the non-smoothness of the solution of (6) does not affect control performance.

Consider now the vector $\hat{\zeta}_i = (y, u, \hat{v}_i)$ with $\hat{v}_i = \beta_i + f_i$, consisting of the signals generated by the potential loop (P/C_i) in response to β_i as shown in Fig. 1. Then, the discrepancy between (P/C_i) and (M_i/C_i) can be measured by computing the outputs $\zeta_i = (y_i, u_i, v_i)$ of the reference loop (M_i/C_i) in response to the same input β_i, and then considering the test functionals

$$J_i(t) = \max_{\tau \leq t} \frac{\|[\Psi(\hat{\zeta}_i - \zeta_i)]^\tau\|_2}{\|\mathcal{F}(\beta_i^\tau)\|_2 + \mu} \tag{8}$$

Please cite this article in press as: Nouri Manzar, M., et al., Input-constrained multi-model unfalsified switching control. Automatica (2017), http://dx.doi.org/10.1016/j.automatica.2017.04.044