Automatica 81 (2017) 1–7

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Distributed average tracking for double-integrator multi-agent systems with reduced requirement on velocity measurements☆

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ARTICLE INFO

Article history:
Received 22 April 2015
Received in revised form
19 December 2016
Accepted 13 February 2017
Available online 19 April 2017

Keywords:
Distributed average tracking
Cooperative control
Multi-agent systems

ABSTRACT

This paper addresses distributed average tracking for a group of physical double-integrator agents under an undirected graph with reduced requirement on velocity measurements. The idea is that multiple agents track the average of multiple time-varying input signals, each of which is available to only one agent, under local interaction with neighbors. We consider two cases. First, a distributed algorithm and filter are proposed, where each agent needs its own and neighbors' filter outputs obtained through communication besides its local relative positions and its input signal, input velocity and input acceleration. Here, the requirement for either absolute or relative velocity measurements is removed. The algorithm is robust to initialization errors and can deal with a wide class of input signals with bounded deviations in input signals, input velocities, and input accelerations. Second, a distributed algorithm and filter are proposed to remove the requirement for communication. Here, each agent needs to measure the relative positions between itself and its neighbors and its own velocity. However, the requirement for relative velocity measurements between the agent and its neighbors is removed. The algorithm is robust to initialization errors and can deal with the case, where the input signals, input velocities and input accelerations are all bounded.

1. Introduction

This paper studies the following distributed average tracking problem: given a group of agents and one time-varying input signal per each agent, design a control law for the agents based on local information such that all the agents will finally track the average of these input signals. The problem has found applications in distributed sensor fusion (Spanos, Olfati-Saber, & Murray, 2005a), feature-based map merging (Aragues, Cortes, & Sagues, 2012), and distributed Kalman filtering (Bai, Freeman, & Lynch, 2011), where the scheme has been mainly used as an estimator. However, there are some applications such as region following formation control (Cheah, Hou, & Slotine, 2009), coordinated path planning (Švestka & Overmars, 1998) or distributed convex optimization (Rahili, Ren, & Lin, 2017) that require the agents' physical states instead of estimator states to converge to a time-varying network quantity, where each agent only has a local and incomplete copy of that quantity. Compared with the consensus and distributed tracking problems, distributed average tracking poses more theoretical challenges, since the tracking objective is time-varying and is not available to any agent.

In the literature, linear distributed algorithms have been employed for special kinds of time-varying input signals. Ref. Spanos, Olfati-Saber, and Murray (2005b) uses frequency domain analysis to study consensus on the average of multiple input signals with steady-state values. In Freeman, Yang, and Lynch (2006), a proportional algorithm and a proportional–integral algorithm are proposed to achieve the distributed average tracking with bounded tracking error, where accurate estimator initialization is relaxed

☆ This research was supported by the National Science Foundation under Grant CMMI-1537729, National Natural Science Foundation of China under Grants 61528301 and 61473240, and the Technology Transformation Program of Chongqing Higher Education University under Grant KJZH17102. The material in this paper was partially presented at the 2015 American Control Conference, July 1–3, 2015, Chicago, IL, USA. This paper was recommended for publication in revised form by Associate Editor Dimos V. Dimarogonas under the direction of Editor Christos G. Cassandras.

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http://dx.doi.org/10.1016/j.automatica.2017.02.043

in the proportional–integral algorithm. In Bai, Freeman, and Lynch (2010), the internal model principle is employed to extend the proportional–integral algorithm to a special group of time-varying input signals with a common denominator in their Laplace transforms, where the denominator also needs to be used in the estimator design. In Kia, Cortes, and Martinez (2013), the authors propose 1st-order-input and 2nd-order-input consensus algorithms to allow the agents to track the average of their dynamic input signals with a pre-specified rate, where the interaction is described by a strongly connected and weight-balanced directed graph. Ref. Zhu and Martinez (2010) addresses discrete-time distributed average tracking of time-varying input signals whose nth order difference is bounded with a bounded error. However, linear algorithms cannot ensure distributed average tracking for general input signals. Therefore, some researchers employ nonlinear tracking algorithms. In Nosrati, Shafiee, and Menhaj (2012), a class of nonlinear algorithms is proposed for input signals with bounded deviations, where the tracking error is proved to be bounded. A nonsmooth algorithm is proposed in Chen, Cao, and Ren (2012), which is able to track time-varying input signals with bounded derivatives.

All the above references primarily study the distributed average tracking problem from a distributed estimation perspective, where the agents implement local estimators through communication with neighbors freely without the need for obeying certain physical agent dynamics. However, there are applications (Cheah et al., 2009; Rahili et al., 2017; Švestka & Overmars, 1998), where the distributed average tracking problem is relevant for designing distributed control laws for physical agents. In these applications, the dynamics of the physical agents must be taken into account in the control law design and the dynamics themselves introduce further challenges to the distributed average tracking problem. Distributed average tracking for physical agents with double-integrator dynamics and general linear dynamics are studied in, respectively, Chen and Ren (2013), Chen, Ren, Lan, and Chen (2015) and Zhao, Duan, and Li (2017). In Chen, Feng, Liu, and Ren (2015) a proportional–integral control scheme is extended to achieve distributed average tracking for physical Euler–Lagrange systems for two different kinds of input signals with steady states and with bounded derivatives.

It is noted that in the existing distributed average tracking algorithms employed for applications with physical double-integrator agents, both relative position and relative velocity measurements are required in the control laws (Chen, Feng et al., 2015; Chen & Ren, 2013; Chen, Ren et al., 2015; Rahili et al., 2017; Zhao et al., 2017). However, in practice, velocity measurements are usually less accurate and more expensive than position measurements. In addition, relative velocity measurements are often more challenging and expensive than absolute velocity measurements. We are hence motivated to solve the distributed average tracking problem for physical double-integrator agents with reduced requirement on velocity measurements, expanding on our preliminary work reported in Ghapani, Ren, and Chen (2015). In the context of distributed average tracking, reducing velocity measurements poses significant theoretical challenges. The reason is that unlike the consensus problem or the single-leader coordinated tracking problem where the leader has access to the tracking objective, there are significant additional inherent challenges in distributed average tracking as none of the agents has access to the tracking objective.

In this paper, two distributed algorithms (controller design combined with filter design) are introduced to achieve distributed average tracking with reduced requirement on velocity measurements and in the absence of correct position and velocity initialization. Each algorithm has its own relative benefits and is feasible for different application scenarios. In the first algorithm design, there is no need for either absolute or relative velocity measurements. Each agent's algorithm employs its local relative positions with respect to neighbors, its own and neighbors' filter outputs accessed through communication and its own input signal, input velocity and input acceleration. The algorithm allows the agents to track the average of a wide class of time-varying input signals with bounded deviations among the input signals, among the input velocities, and among the input accelerations. Using this algorithm, distributed average tracking can be achieved in the absence of velocity measurements and correct initialization. In the second algorithm design, there is still no requirement for correct initialization and relative velocity measurements. Furthermore, inter-agent communication is not necessary and the algorithm can be implemented using only local sensing, which is desirable for certain applications (e.g., deep-space spacecraft formation flying), where communication might not be desirable or available. Each agent's algorithm only employs its local relative positions with respect to neighbors, its own velocity, input signal, input velocity and input acceleration. Distributed average tracking can be achieved provided that the input signals and their velocities and accelerations are all bounded.

Notations: Throughout the paper, $\mathbb{R}$ denotes the set of all real numbers and $\mathbb{R}^+$ the set of all positive real numbers. Let $\mathbf{1}_n$ and $\mathbf{0}_n$ denote the $n \times 1$ column vector of all ones and all zeros respectively. Let $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote, respectively, the maximum and minimum eigenvalues of a square real matrix with real eigenvalues. We use $\otimes$ to denote the Kronecker product, and $\mathrm{sgn}(\cdot)$ to denote the signum function defined componentwise. For a vector function $x(t) : \mathbb{R} \mapsto \mathbb{R}^m$, define $\|x(t)\|_p$ as the p-norm, $x(t) \in \mathbb{L}_2$ if $\int_0^\infty x(\tau)^T x(\tau)\mathrm{d}\tau < \infty$ and $x(t) \in \mathbb{L}_\infty$ if for each element of $x(t)$, $\sup_{t\geq 0} |x_i(t)| < \infty, i = 1, \ldots, m$.

2. Problem statement

Here, we consider n physical agents described by double-integrator dynamics

$$\dot{x}_i(t) = v_i(t), \qquad \dot{v}_i(t) = u_i(t), \quad i = 1, \ldots, n, \tag{1}$$

where $x_i(t) \in \mathbb{R}^p$ and $v_i(t) \in \mathbb{R}^p$ are, respectively, ith agent's position and velocity, and $u_i(t)$ is its control input.

An *undirected* graph $G \triangleq (V, E)$ is used to characterize the interaction topology among the agents, where $V \triangleq \{1, \ldots, n\}$ is the node set and $E \subseteq V \times V$ is the edge set. An edge $(j, i) \in E$ means that node i can obtain information from node j and vice versa. Self edges (i, i) are not considered here. N_i is the set of agents that are neighbors of agent i. Let m denote the number of edges in E, where the edges (j, i) and (i, j) are counted only once. The set of neighbors of node i is denoted as N_i. The *adjacency matrix* $\mathbf{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ of the graph G is defined such that the edge weight $a_{ij} = 1$ if $(j, i) \in E$ and $a_{ij} = 0$ otherwise. For an undirected graph, $a_{ij} = a_{ji}$. The *Laplacian matrix* $L = [l_{ij}] \in \mathbb{R}^{n\times n}$ associated with $\mathbf{A}$ is defined as $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \neq j$. For an undirected graph, L is symmetric positive semi-definite. By arbitrarily assigning an orientation for the edges in G, let $D \triangleq [d_{ij}] \in \mathbb{R}^{n\times m}$ be the *incidence matrix* associated with G, where $d_{ij} = -1$ if the edge e_j leaves node i, $d_{ij} = 1$ if it enters node i, and $d_{ij} = 0$ otherwise. The Laplacian matrix L is then given by $L = DD^T$ (Royle & Godsil, 2001).

Assumption 2.1. The undirected graph G is connected.

Lemma 2.1 (*Royle & Godsil, 2001*)**.** *Under Assumption 2.1, the Laplacian matrix L has a simple zero eigenvalue such that $0 = \lambda_1(L) < \lambda_2(L) \leq \cdots \leq \lambda_n(L)$, where $\lambda_i(\cdot)$ denotes the ith eigenvalue. Furthermore, for any vector $y \in \mathbb{R}^n$ satisfying $\mathbf{1}_n^T y = 0$, $\lambda_2(L)y^T y \leq y^T Ly \leq \lambda_n(L)y^T y$.*

Suppose that each agent has a time-varying input signal $x_i^r(t) \in \mathbb{R}^p$, $i = 1, \ldots, n$, satisfying

$$\dot{r}_i(t) = v_i^r(t), \qquad \dot{v}_i^r(t) = a_i^r(t), \tag{2}$$