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Brief paper

Stability of decentralized model predictive control of graph-based power flow systems via passivity☆

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ABSTRACT

This work presents a passivity-based stability guarantee for the decentralized control of nonlinear power flow systems. This class of systems is characterized using a graph-based modeling approach, where vertices represent capacitive elements that store energy and edges represent power flow between these capacitive elements. Due to their complexity and size, these power flow systems are often decomposed into dynamically coupled subsystems, where this coupling stems from the exchange of power between subsystems. Each subsystem has a corresponding model predictive controller that can be part of a decentralized, distributed, or larger hierarchical control structure. By exploiting the structure of the coupling between subsystems, stability of the closed-loop system is guaranteed by augmenting each model predictive controller with a local passivity constraint.

1. Introduction

Decentralized control of large systems comprised of dynamically coupled subsystems spans many application areas including thermal systems (Chandan, 2013; Jain, Koeln, Sundaram, & Alleyne, 2014; Morosan, Bourdais, Dumur, & Buisson, 2010), water distribution networks (Cantoni et al., 2007; Negenborn, Sahin, Lukszo, De Schutter, & Morari, 2009; Ocampo-Martinez, Barcelli, Puig, & Bemporad, 2012), chemical process networks (Christofides, Scattolini, de la Peña, & Liu, 2013; Tippett & Bao, 2012), microgrids (Guerrero, Chandorkar, Lee, & Loh, 2013; Riverso, Farina, & Ferrari-Trecate, 2013; Zamora & Srivastava, 2010), and flow networks (Bauso, Blanchini, Giarre, & Pesenti, 2013; Blanchini, Franco, Giordano, Mardanlou, & Montessoro, 2016). These applications, characterized by the flow and conservation of a resource, can be considered cases of a larger class of power flow systems. Power flow systems are governed by the transportation, conversion, and storage of energy across domains. Graph-based system representation is a widely adopted modeling technique that readily captures the structure of the governing mass and energy conservation laws for these systems (Blanchini et al., 2016; Heo, Rangarajan, Daoutidis, & Jogwar, 2011; Moore, Vincent, Lashhab, & Liu, 2011; Preisig, 2009). Vertices, or nodes, represent capacitive elements that store energy, and edges represent power flow paths between these capacitive elements. While local parameters and functional relationships for power flow depend on the energy domain, system structure, analysis, and control are energy domain agnostic. This makes a graph-based approach a powerful tool for the modeling and control of a complex system-of-systems, comprised of multiple systems with various energy domains.

Additionally, the governing energy conservation laws suggest another unifying inherent feature of these systems: passivity. The notion of passivity in system modeling and control originated from the physical principles of energy conservation and dissipation in electrical and mechanical systems (Hill & Moylan, 1976) and has become a widely used and highly general methodology in nonlinear system analysis and control (Khalil, 2002; Sepulchre, Jankovic, & Kokotovic, 1997; van der Schaft, 1996). Thus, passivity-based control has been applied to a variety of power flow systems in centralized (Mukherjee, Mishra, & Wen, 2012; Ortega, Loria, Nicklasson, & Sira-Ramirez, 1998; Ulbig, 2007) and decentralized control architectures (Bao & Lee, 2007).

Model Predictive Control (MPC) is well suited for controlling power flow systems. The ability to account for actuator and state

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constraints and utilize communication and disturbance preview information allows MPC to maximize the performance and efficiency of these systems. Centralized, passivity-based, MPC has been implemented in Falugi (2014), Løvaas, Seron, and Goodwin (2007), Raff, Ebenbauer, and Allgöwer (2007), Sredojev and Eaton (2014) and Yu, Zhu, Xia, and Antsaklis (2013). Decentralized passivity-based MPC extends this approach to systems with a large number of states and actuators (Tippett & Bao, 2012; Varutti, Kern, & Findeisen, 2012). In these approaches, along with those developed in Arcak and Sontag (2008) and Yu and Antsaklis (2010), stability is assessed with a global, system-wide matrix condition that accounts for the subsystem interconnection topology and the gain of the coupling between subsystems.

The aim of this paper is to present a purely decentralized and easily implementable method for augmenting existing decentralized control frameworks that guarantees stability of the overall closed-loop system. The relative simplicity of the approach is enabled by focusing on the control of power flow systems represented as graphs. The proposed approach identifies a set of inputs and outputs that render each subsystem passive. Neighboring subsystems form a negative feedback connection, establishing passivity of the overall system. While the approach relies on a graph-based representation of the system, a nonlinear, affine in control, power flow representation provides applicability to a wide class of systems. Actuator input and state constraints are considered, with slack variables on the state constraints to avoid infeasibility issues. Through the addition of a nonlinear constraint to each controller, the proposed approach provides simple implementation and reduced conservatism compared to standard passivity-based approaches.

The remainder of this paper is organized as follows. Section 2 introduces the graph-based modeling framework for the class of power flow systems. Section 3 presents the main results of the paper including establishing passivity of individual subsystems, analyzing the passivity-preserving interconnections between subsystems, developing a passivity constraint for each MPC controller, and proving the stability of the closed-loop system. Concluding remarks are provided in Section 4.

1.1. Notation

The symbol $\mathbb{R}$ denotes the set of real numbers. For the scalar function $f(x)$, $\mathcal{N}(f(x)) = \{x|f(x) = 0\}$ denotes the zero set of $f(x)$. A vector v with elements v_i is defined as $v = [v_i]$. Similarly, a matrix M with elements m_{jk} in the jth row and kth column is defined as $M = [m_{jk}]$. The eigenvalues of matrix $A \in \mathbb{R}^{n\times n}$ are $\lambda_k(A)$, $k \in [1, n]$ and their real parts are denoted $\operatorname{Re}\lambda_k(A)$, $k \in [1, n]$.

2. Class of systems

Consider a power flow system composed of N interconnected subsystems $\mathbf{S}_i$, $i \in [1, N]$. Each subsystem is represented by an oriented graph $\mathcal{G}_i = (V_i, E_i)$ with the set of vertices V_i and set of edges E_i. Each oriented edge $e_{i,j} \in E_i$ represents power flow in $\mathbf{S}_i$, where positive power $P_{i,j}$ flows from the tail vertex $v^{tail}_{i,j}$ to the head vertex $v^{head}_{i,j}$. Each vertex $v_{i,k} \in V_i$ has an associated state $x_{i,k}$ that represents the energy stored in that vertex. Thus, the dynamic of each vertex $v_{i,k}$ satisfies the energy conservation equation

$$C_{i,k}\dot{x}_{i,k} = \sum_{e_{i,j}\in E^{in}_{i,k}} P_{i,j} - \sum_{e_{i,j}\in E^{out}_{i,k}} P_{i,j}, \tag{1}$$

where $C_{i,k} > 0$ is the energy storage capacitance of vertex $v_{i,k}$ and $E^{in}_{i,k}$ and $E^{out}_{i,k}$ represent the sets of edges oriented into and out of vertex $v_{i,k}$.

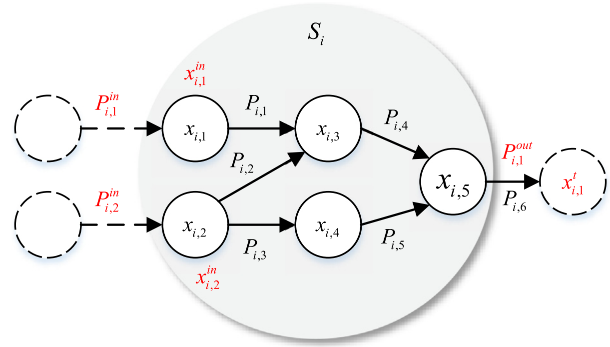

Fig. 1. Notional subsystem exemplifying the graph-based power flow representation with key power flows and states highlighted in red. Dashed lines indicate elements that serve as disturbances to the subsystem. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Assumption 1. The power flow $P_{i,j}$ along edge $e_{i,j}$ is defined as

$$P_{i,j} = f_{i,j}(x^{tail}_{i,j}, x^{head}_{i,j}) + g_{i,j}(x^{tail}_{i,j}, x^{head}_{i,j})u_{i,j}, \tag{2}$$

where $x^{tail}_{i,j}$ and $x^{head}_{i,j}$ are the states of the tail and head vertices $v^{tail}_{i,j}$ and $v^{head}_{i,j}$, $u_{i,j}$ is an associated actuator input, and $f_{i,j}, g_{i,j} : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$. Additionally, $f_{i,j}$ is Lipschitz, twice continuously differentiable, and $f_{i,j}(0, 0) = 0$ while $g_{i,j}$ is continuous, $g_{i,j}(0, 0) = 0$, and the intersection of the zero sets of $g_{i,j}$ is the origin, $\bigcap_j \mathcal{N}\left(g_{i,j}(x^{tail}_{i,j}, x^{head}_{i,j})\right) = \{0\}$.

Fig. 1 shows a graph of an example subsystem $\mathbf{S}_i$ used to identify key components. For this example subsystem, there are three paths for power to enter or exit the subsystem. For the two dashed edges oriented into the subsystem, the power flow along these edges, denoted $P^{in}_{i,1}$ and $P^{in}_{i,2}$, is treated as a disturbance to the subsystem and these edges are not included in $\mathcal{G}_i$. The third path is represented by an edge oriented out of the subsystem, labeled $P^{out}_{i,1}$. Power flow along this type of edge follows the relationship from (2), where now $x^{head}_{i,j}$ is a sink vertex state $x^t_{i,1}$. These sink states are *not* states of $\mathbf{S}_i$ and thus are disturbances to the subsystem, representing the surrounding environment. Finally, as indicated in Fig. 1, each subsystem has a subset x^{in}_i of the states x_i that represent the states directly affected by the inlet power flows P^{in}_i.

Let $M_i = [m_{i,jk}]$ be the incidence matrix of graph $\mathcal{G}_i$ (West, 2001) where

$$m_{i,jk} = \begin{Bmatrix} +1 & v_{i,j} \text{ is the tail of } e_{i,k} \\ -1 & v_{i,j} \text{ is the head of } e_{i,k} \\ 0 & \text{else} \end{Bmatrix}. \tag{3}$$

Then, based on (1), the subsystem dynamics are

$$\begin{bmatrix} C_i\dot{x}_i \\ \dot{x}^t_i \end{bmatrix} = -M_iP_i + \begin{bmatrix} D_i \\ 0 \end{bmatrix} P^{in}_i, \tag{4}$$

where x_i are the states of the dynamic vertices, x^t_i are the states of the sink vertices, $C_i = \operatorname{diag}([C_{i,k}])$ is a diagonal matrix of the capacitances of the dynamic vertices, P_i are the power flows along the edges of $\mathcal{G}_i$, P^{in}_i are the source power flows entering $\mathbf{S}_i$, and $D_i = [d_{i,jk}]$ is a matrix where

$$d_{i,jk} = \begin{Bmatrix} 1 & v_{i,j} \text{ is the head of } P^{in}_{i,k} \\ 0 & \text{else} \end{Bmatrix}. \tag{5}$$

Since x^t_i are disturbances to the subsystem, not states, M_i is partitioned as

$$C_i\dot{x}_i = -\overline{M}_iP_i + D_iP^{in}_i, \quad M_i = \begin{bmatrix} \overline{M}_i \\ \underline{M}_i \end{bmatrix}. \tag{6}$$