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Brief paper

Foraging motion of swarms with leaders as Nash equilibria☆

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ABSTRACT

The consequences of having a leader in a swarm are investigated using differential game theory. We model foraging swarms with leader and followers as a non-cooperative, multi-agent differential game. The agents in the game start from a set of initial positions and migrate towards a target. The agents are assumed to have no desire, partial desire or full desire to reach the target. We consider two types of leadership structures, namely hierarchical leadership and a single leader. In both games, the type of leadership is assumed to be passive. We identify the realistic assumptions under which a unique Nash equilibrium exists in each game and derive the properties of the Nash solutions in detail. It is shown that having a passive leader economizes in the total information exchange at the expense of aggregation stability in a swarm. It turns out that, the leader is able to organize the non-identical followers into harmony under missing information.

1. Introduction

There are certain advantages of having a leader in a swarm. The leader may initiate the route and the remaining group members follow that path (Estrada & Vargas-Estrada, 2013). Therefore, leader designates the search direction (Wang & Wang, 2008). By leader guidance, a wider area can be covered and the collisions can be avoided (Wang & Wang, 2008). Moreover, leader–follower swarms reach consensus more rapidly (Estrada & Vargas-Estrada, 2013). There are also cases, where consensus may not even be guaranteed by only simple rules and choices of specific leaders become necessary to ensure consensus (King & Cowlishaw, 2009). Leadership also provides orientation improvement and coordination via communication in the group (Andersson & Wallander, 2004; Weimerskirch, Martin, Clerquin, Alexandre, & Jiraskova, 2001). Leader–follower swarms have a multitude of practical applications such as robot teams, ship flocks, UAVs, and vehicle platoons. The leader may play various roles in such systems. In robot teams, a leader is generally an active one, who itself is motion-controlled by an external control input (Kawashima & Egerstedt, 2014). In ship flocks, leader may enable coordination of possibly under-actuated followers (Lapierre, Soetanto, & Pascoal, 2003). In unmanned aerial vehicles, leader may provide reference position and velocity for followers (Karimoddini, Lin, Chen, & Lee, 2013). In vehicle platoons, leader ensures string stability where tight formations are maintained (Peters, Middleton, & Mason, 2014). In optimization techniques such as PSO, leader usually follows the shortest path, i.e., the line towards the minimum and the followers perform the search around that line (Chatterjee, Goswami, Mukherjee, & Das, 2014). In all these systems, leaders constitute a small subset of the group that guides the coordination of the whole network (Estrada & Vargas-Estrada, 2013).

We strive to understand the mechanisms of spontaneous formation of swarms via dynamic non-cooperative game theory of Basar and Olsder (1995) and necessary conditions of optimality of Kirk (2012). We define "spontaneous formation" as the formation of collective behavior based on non-cooperative decisions. Nash equilibrium is ideally suited to model such mechanisms. In Nash equilibrium, each agent gives a best response to the decisions of other agents which results in a collective behavior. We use a game theoretical model and ask whether such equilibrium exists. It turns out that the Nash solution exists and is unique for continuous strategies and for the information structures studied here.

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The difficulty of establishing the existence of Nash equilibria in dynamic multi-agent games with non-convex cost functions is well known (Bressan & Shen, 2004). This continues the quest in Özgüler and Yıldız (2013) and Yıldız and Özgüler (2015), in which, the existence and uniqueness of two swarm games were successfully shown under some realistic assumptions on the information structure among group members and on the allowed strategies to the agents. Here, we focus on *passive leaders* that *are* singled out by the other agents, not because they command, coordinate, or organize, but because of their present geographical position in the group. We study two information structures that define games with passive leaderships. The first structure corresponds to an "ordered graph", Chvátal (1984), and here it is referred to as *hierarchical leadership*. The second structure corresponds to a "directed star" graph, Colbourn, Hoffman, and Rodger (1991), and here it is referred to as *single leadership*. In both games, the swarm members are allowed to be "non-identical" and each member measures its distance only to those members that are ahead. Both games may be compared with the v-formation of birds (although we limit our study to one-dimensional swarms) because an agent's (level of) leadership depends on how close it is to the top of the hierarchy, Nagy, Ákos, Biro, and Vicsek (2010) and Wang and Wang (2008). These games have a loose information structure as very little amount of attention span is needed from an agent during its journey. One consequence of this sparsity in intra-swarm communication is economy in energy expenditure. Power and energy expenditure reduction is indeed an essential feature of v-formation (Cutts & Speakman, 1994; Hainsworth, 1988; Weimerskirch et al., 2001), and (Speakman & Banks, 1998).

The swarming models introduced in this article offer significant improvements over (Özgüler & Yıldız, 2013; Yıldız & Özgüler, 2015). Current models cover non-identical agents, which extends the identical agent structure of Özgüler and Yıldız (2013) and Yıldız and Özgüler (2015). Also, in the current model, the agents act with position information of only the forward agents. Ordered graph and directed star information structures used here are less restrictive than those in Özgüler and Yıldız (2013) and Yıldız and Özgüler (2015). Note that, neither of the four information structures (the ones here and those in Özgüler and Yıldız (2013) and Yıldız and Özgüler (2015)) is a special case of the remaining three.

The paper is organized as follows. Section 2 contains the definitions of the games considered, the individual cost functions that model the motive of each agent and their interpretation as the total effort of an agent in the foraging journey. In Section 3, the main results, the existence and uniqueness of a Nash solution, and its features that relate to a swarming behavior are listed. In Section 5, we discuss the necessity of the constraints posed in the definitions of the games. In Section 4, four swarm games that have Nash solutions are compared. Section 6 is on conclusions. Detailed proofs of Theorems 1 and 2 are given on the web page (Yıldız & Özgüler, 2016).

2. Two games with leader–follower structure

The games defined are based on motives of a group of agents under two different hypotheses on information structure. In both games, when the agents are assumed to be foraging, say, for food, they start from some initial positions and try to migrate towards a target location. In cases of foraging or non-foraging, and also with or without specified target location, we would like to show that the non-cooperative motives of the agents lead to a collective behavior dictated by a Nash Equilibrium of the games, whenever it exists.

Game L1 (Hierarchical Leadership): Determine $\min_{u^i}\{L^i\}$ *subject to* $\dot{x}^i = u^i,\ \forall i = 1, \ldots, N$, where

$$L^1 := \gamma \frac{x^1(T)^2}{2} + \int_0^T \frac{u^1(t)^2}{2}\, dt,$$
$$L^i := \beta \frac{x^i(T)^2}{2} + \int_0^T \left\{ \frac{u^i(t)^2}{2} + \sum_{j=1}^{i-1} \left(a_j \frac{[x^i(t) - x^j(t)]^2}{2} \right.\right. \tag{1}$$
$$\left.\left. - r_j |x^i(t) - x^j(t)| \right) \right\} dt, \quad 2 \le i \le N.$$

Game L2 (Single Leader): Determine $\min_{u^i}\{L^i\}$ *subject to* $\dot{x}^i = u^i,\ \forall i = 1, \ldots, N$, where

$$L^1 := \gamma \frac{x^1(T)^2}{2} + \int_0^T \frac{u^1(t)^2}{2}\, dt,$$
$$L^i := \beta \frac{x^i(T)^2}{2} + \int_0^T \left\{ \frac{u^i(t)^2}{2} + \bar{a}_i \frac{[x^i(t) - x^1(t)]^2}{2} \right. \tag{2}$$
$$\left. - \bar{r}_i |x^i(t) - x^1(t)| \right\} dt, \quad 2 \le i \le N.$$

In both games, L^1 is the cost minimized by one agent and $L^i,\ i = 2, \ldots, N$, are the costs minimized by the others, where N is the *swarm population*. The swarming duration is specified as $T > 0$, $u^i(t) = \dot{x}^i$ is the control input, and $x^i(t)$ is the position at time $t \in [0, T]$ of the ith agent. The *adhesion* $a_j > 0$ is an attraction parameter and $r_j > 0$ is a repulsion parameter. Parameters $\gamma \ge 0$ and $\beta \ge 0$ weigh the foraging efforts; the higher they are, the better is the desire to reach foraging target by the respective agent. The agents control their velocities to minimize their total effort, which consists of kinetic energy $u^i(t)^2$ as well as the artificial potential energy. Here, combined attractive, repulsive, and foraging terms in the cost function of an agent is interpreted as the artificial potential energy of that agent, Gazi and Passino (2004).

The exact foraging target is normalized to be the origin in $x^1(T) \ldots x^N(T)$-space. The agents may have varying degrees of desires to reach this target in Games L1 and L2. The foraging task is performed through the presence of the foraging terms with weights γ and β in the cost functions since their minimization will imply that an agent is as close to the origin as possible. If these terms are removed from the cost functions and, instead, the terminal conditions $x^1(T) = 0, \ldots, x^N(T) = 0$ are required, then this is a slightly different game and will be referred to as the *specified terminal condition* game. If $x^1(T), \ldots, x^N(T)$ are altogether free, then there is no foraging requirement and the corresponding slightly different games (in which the foraging terms are simply removed from the cost functions) will be called the *free terminal condition* games.

The cost functions considered in this game are similar to those in Özgüler and Yıldız (2013) and Yıldız and Özgüler (2015) with important differences. In all games, the indexing of the agents indicate the ranking in the initial queue of the agents. The agent of index 1 starts at the closest position to the foraging target and that with index N, to be at the farthest. Here, agent-1 and others have different cost function structures, as opposed to the uniform structure in Yıldız and Özgüler (2015). Second, we extend the identical agent form of Yıldız and Özgüler (2015) to non-identical agents by allowing coefficients a and r to vary among different agents who have no desire, partial desire, or full desire to reach the target. Above all, we alter the self organized structure in Özgüler and Yıldız (2013) and Yıldız and Özgüler (2015) to a leader–follower structure. The agent of index 1 is distinguished by its ignorance of the position of any other member in the group in the duration of the whole journey. Each agent in Game L1 is assumed to observe (measure) and know the positions of the agents ahead of