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Brief paper

Two-time-scale adaptive internal model designs for motion coordination☆

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ABSTRACT

We study a motion coordination problem where a group of agents is required to reach consensus while tracking a leader's sinusoidal reference velocity. We assume that the frequency of the reference velocity is available only to the leader. Building on a passivity-based control, we develop decentralized two-time-scale adaptive internal model control algorithms that estimate the unknown frequency information and achieve consensus of the group. We establish exponential stability of the algorithms using two-time-scale averaging theory. Simulation results with first order and second order agent dynamics illustrate the effectiveness of the proposed controls.

1. Introduction

The internal model principle (IMP) (Francis & Wonham, 1976) is a fundamental control design tool for rejecting disturbances and tracking signals generated by exogenous systems. Recently, the IMP has been employed to design cooperative control laws for multi-agent systems. In Bai, Arcak, and Wen (2011), a connection between the IMP and passivity was established to achieve adaptive motion coordination. The IMP has also been used in output regulation (Huang, 2015), dynamic average consensus (Bai, Freeman, & Lynch, 2010), distributed Kalman filtering (Bai, Freeman, & Lynch, 2011), and synchronization (De Persis & Jayawardhana, 2012; Shafi & Bai, 2015; Wieland, Sepulchre, & Allgöwer, 2011).

One common assumption in using the IMP is that a model of the exogenous system is available for control designs. When the exogenous system generates a sinusoidal signal, this assumption means that the frequency of the sinusoid is known. In certain scenarios, prior knowledge of the exogenous system can be identified or obtained (e.g., in controlled environments) and thus this assumption is valid. However, it restricts the application of the IMP in scenarios where the model of the exogenous system is unknown or changes over time. For example, autonomous agents may update its reference velocity in reaction to dynamic environments. Therefore, the model information of the reference velocity may change over time.

In this paper, we consider a group of agents whose objective is to reach consensus while tracking a leader's sinusoidal reference velocity subject to a constant bias. A sinusoidal velocity represents the typical motion when a vehicle orbits. For example, when an aircraft makes a coordinated turn, its linear velocity is a sinusoidal function with the frequency being the angular rate of the turn. In the presence of constant disturbances, such as wind, the linear velocity becomes biased sinusoids.

When the frequency of the reference velocity is known, Bai et al. (2011, Chapter 3) provided a passivity-based internal model control design that achieves consensus and tracking of the reference velocity. We extend the design in Bai et al. (2011) to the scenario where the frequency of the sinusoidal reference velocity is unknown to the followers. One approach for this scenario is to parameterize the internal model and estimate the unknown parameters in the internal model (Marino & Tomei, 2003; Nikiforov, 2001; Serrani, Isidori, & Marconi, 2001; Su & Huang, 2013). However, this parameter estimation approach requires re-designing the passivity-based control and does not exploit a passivity property in the internal model.

We take an approach different from the parameter estimation approach. We develop a two-time-scale adaptive internal model control approach that takes advantage of the passivity properties inherent in the internal model, the agent dynamics, and the

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network, and yields decentralized control algorithms. In particular, the two-time-scale approach consists of a slow system estimating the frequency of the reference velocity and a fast system representing the group dynamics. The stability of the fast system is ensured by the passivity properties when the agents' estimated frequencies are treated as frozen parameters. As the transients of the group dynamics vanish, the slow system becomes periodic, allowing us to analyze its stability using averaging theory. With the stability of the fast system and the slow system established, we use two-time-scale averaging theory (Sastry & Bodson, 1989) to prove the convergence of the estimated frequency to the true frequency and consensus of the agents. Our approach estimates only the frequency of the sinusoid and does not require re-designing the passivity-based control law.

We consider two scenarios for the two-time-scale approach. In the first scenario, each follower updates its frequency estimate without communicating with other followers. In the second scenario, we allow each follower to communicate its local control signal to its neighbors. We augment the control design in the first scenario with a coupling term that combines the control signals from the neighboring agents. Using numerical examples, we show that the coupling term becomes an additional degree of freedom to tune the system and results in improved transient performance.

The main contribution of this paper is a two-time-scale averaging approach to designing *decentralized adaptive* internal model controls for multi-agent systems with passive dynamics. This approach enables agents to achieve consensus while tracking a completely unknown sinusoidal reference velocity subject to a constant bias. We note that Li, Liu, Ren, and Xie (2013) studied distributed tracking control with a bounded reference velocity and proposed discontinuous distributed control laws to achieve consensus. However, Li et al. (2013) assume that agents' dynamics are identical and accurately known for LMI-based control designs. Since our approach employs input–output properties of the multi-agent system, it allows heterogeneous and unknown agent dynamics. We also note that similar frequency estimation approaches for a single system have been discussed in Esbrook, Tan, and Khalil (2011) and Brown and Zhang (2004).

The rest of the paper is organized as follows. In Section 2, we review a passivity-based design when the frequency of the reference velocity is available to all the agents. In Section 3, we consider the scenario where the frequency of the reference velocity is available only to the leader and present our two-time-scale adaptive design. In Section 4, we analyze the stability of the proposed adaptive design. In Section 5, we introduce an augmented adaptive control design when the followers are allowed to communicate their local control signals. Section 6 presents two simulation examples to illustrate the effectiveness of the designs. Conclusions and future work are discussed in Section 7.

Notation: The vectors 1_N and 0_N represent the N by 1 vectors with all entries 1 and 0, respectively. The set of real numbers is denoted by $\mathbb{R}$. The set of N by M real matrices is denoted by $\mathbb{R}^{N\times M}$. Let I_N be the $N\times N$ identity matrix. The Kronecker product of matrices A and B is denoted by $A\otimes B$. The notation $\mathrm{diag}\{A_1,\ldots,A_n\}$ denotes a block diagonal matrix with A_i on the diagonal, where A_i can be a matrix or a scalar. The transpose of a real matrix A is denoted by A^T. We denote by $L_{i,j}$ the entry in the ith row and the jth column of a matrix L.

2. A cooperative system

We consider a cooperative system of N agents, where the state of each agent $i, i=1,\ldots,N$, is represented by $x_i\in\mathbb{R}^p$. To simplify the notation we consider the scalar case $p = 1$. However, the results in this paper extend to $p > 1$ with the use of Kronecker algebra. The information flow between the agents is described by a connected and bidirectional graph $\mathcal{G}$. Let $\mathcal{N}_i$ denote the set of neighbors of agent i. We define a weighted graph Laplacian matrix L of $\mathcal{G}$, whose elements are given by

$$L_{i,j}=\begin{cases}\sum_{\forall j} a_{ij} & i=j\\ -a_{ij} & i\neq j,\end{cases} \tag{1}$$

where $a_{ij}=a_{ji}\geq 0$ and $a_{ij}>0$ if and only if $j\in\mathcal{N}_i$. Since $\mathcal{G}$ is undirected, L is symmetric and satisfies $1_N^TL=0_N$ and $L1_N=0_N$.

Our objective is to design decentralized algorithms such that x_i's asymptotically reach consensus and $\dot{x}_i$'s follow a predefined reference velocity $v(t)$, i.e.,

$$|x_i-x_j|\to 0,\quad \forall i,j \tag{2}$$

and

$$|\dot{x}_i-v(t)|\to 0,\quad \forall i. \tag{3}$$

The reference velocity $v(t)$ dictates the translation of the group motion. For a constant $v(t)$, the group motion translates along a straight line. In this paper, we consider a sinusoidal $v(t)$ of frequency ω, which can represent periodic motions of the group. In Section 3, we show that the proposed adaptive designs can be extended to handle a sinusoidal $v(t)$ with a constant bias.

We can write any sinusoidal reference velocity $v(t)$ as an output of a dynamical system G_1 given by

$$G_1:\begin{cases}\dot{\xi}_1=A\xi_1\\ v(t)=b^T\xi_1,\end{cases} \tag{4}$$

where

$$A=\begin{bmatrix}0 & -\omega\\ \omega & 0\end{bmatrix},\quad \omega>0, \tag{5}$$

and

$$b^T=[0\quad 1]. \tag{6}$$

Different initial conditions of ξ_1 give rise to sinusoidal $v(t)$ with different magnitudes and phases.

We assume that the agent dynamics, through a coordinate transformation, can be brought to the following form:

$$\dot{x}_i=y_i+v(t) \tag{7}$$

$$\begin{aligned}\dot{\zeta}_i&=F_i\zeta_i+H_i(-u_i)\\ y_i&=C_i\zeta_i,\end{aligned} \tag{8}$$

where u_i is defined as

$$u_i=\sum_{j\in\mathcal{N}_i} a_{ij}(x_i-x_j). \tag{9}$$

Reference Arcak (2007) showed that (7)–(9) achieve (2) and (3) if H_i is full column rank and if the ζ_i subsystem is strictly passive from $-u_i$ to y_i, that is, there exists a positive definite storage function $W_i(\zeta_i)$ such that

$$\dot{W}_i\leq -u_iy_i-k_i|\zeta_i|^2,\quad k_i>0. \tag{10}$$

The design in (7) and (8) assumes that every agent has the $v(t)$ information. This assumption was relaxed in Bai et al. (2011) to the case where $v(t)$ is available only to a leader, say the first agent, and the other agents only have the frequency information of $v(t)$. Bai et al. (2011) provided a passivity-based adaptive design that recovers (2) and (3). This adaptive design starts with replacing the $v(t)$ in (7) by $\hat{v}_i$, an estimate of $v(t)$ for agent i, $i=2,\ldots,N$. Then the estimate $\hat{v}_i$ is updated based on an internal model control. The resulting closed-loop system is given by

$$\begin{aligned}\dot{x}_1&=y_1+v(t)\\ \dot{x}_i&=y_i+\hat{v}_i,\quad i=2,\ldots,N,\end{aligned} \tag{11}$$