Automatica 75 (2017) 293–298

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Consensus conditions for general second-order multi-agent systems with communication delay☆

Wenying Hou [a], Minyue Fu [b,c], Huanshui Zhang [a], Zongze Wu [c]

[a] School of Control Science and Engineering, Shandong University, Jinan 250061, China

[b] School of Electrical Engineering and Computer Science, University of Newcastle, NSW 2308, Australia

[c] School of Automation, Guangdong University of Technology, Guangdong Key Laboratory of IoT Information Technology, Guangzhou 510006, China

ARTICLE INFO

Article history:
Received 27 October 2015
Received in revised form
20 July 2016
Accepted 26 August 2016

Keywords:
Time-delay systems
Multi-agent systems
Frequency domain method
Second-order consensus

ABSTRACT

This paper studies the consensus problem for a class of general second-order multi-agent systems (MASs) with communication delay. We first consider the delay-free case and obtain a necessary and sufficient condition for consensus. Then, based on the obtained conditions for the delay-free case, we deduce an explicit formula for the delay margin of the consensus for the case with time delay using the relationship between the roots of the characteristic equation and the time delay parameter. In addition, we consider the special case where the second-order model is a double integrator. For this case, simpler consensus conditions under communication delay are provided.

1. Introduction

Network consensus is a fundamental distributed control and optimization problem. After a couple of decades of active research on network consensus, it is well recognized by now that consensus control finds wide applications in areas including multi-agent coordination (such as coordinated decision making (Bauso, Giarre, & Pesenti, 2003), vehicle formations (Fax & Murray, 2004), rendezvous problem (Lin, Morse, & Anderson, 2003), distributed computation (Lynch, 1997), and flocking (Olfati-Saber, 2006), et al.), smart electricity networks (Ma, Chen, Huang, & Meng, 2013) and biological group behavioral analysis (Strogatz, 2001). The key of consensus control is to design an appropriate consensus protocol based on local information exchange such that all the agents (or nodes) in a network agree upon certain quantities of common interest.

The pioneering work of Olfati-Saber and Murray (2004) solved an average consensus problem for first-order integrator networks by using the algebraic graph theory and frequency domain analysis. Since then, there has been a large number of results on consensus, e.g., Avrachenkov, Chamie, and Neglia (2011), Fax and Murray (2004), Moreau (2005), Olfati-Saber, Fax, and Murray (2007) and Ren and Beard (2005). All of the above results on the first-order consensus problems focus on the first-order integrator systems or networks without time delay. However, the conditions that can guarantee consensus for the first-order MASs, for example, the network communication topology has a directed spanning tree, may not ensure the second-order MASs to reach consensus. In addition, in most applications, it is inevitable that time delay exists in the information transmission between agents due to communication congestion and finite transmission bandwidth. The existence of the communication delay will inevitably deteriorate the control performance and stability of a networked control system. Therefore it is important to consider consensus conditions of higher order MASs with communication delay.

Although there have been several papers studying the consensus problem with time delay, such as Hou, Fu, and Zhang (2016), Wang, Saberi, Stoorvogel, Grip, and Yang (2013), Wang, Xu, and Zhang (2014) and Xu, Zhang, and Xie (2013), they only focus on first-order consensus or they cannot give the explicit formula for the time delay margin for achieving consensus. Middleton and Miller (2007) considered time delay margin for unstable plants using frequency domain analysis. Second-order consensus problems can model more realistic dynamics of MASs. As far as the authors know, there are few papers considering the consensus problem for

☆ This work is supported by the National Science Foundation of China under Grants 61120106011,61573221. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Wei Ren under the direction of Editor Christos G. Cassandras.

E-mail addresses: wyhou_00@163.com (W. Hou), Minyue.fu@newcastle.edu.au (M. Fu), hszhang@sdu.edu.cn (H. Zhang), zzwu@scut.edu.cn (Z. Wu).

http://dx.doi.org/10.1016/j.automatica.2016.09.042
0005-1098/

general second-order dynamic systems with time delay. Ren and Atkins (2007) and Yu, Chen, and Cao (2010) considered the second-order consensus problem but only focused on double integrator systems.

In this paper, we consider the consensus condition for a class of MASs which contain a general second-order linear dynamic model for each agent and involve communication delay between agents. We first obtain a necessary and sufficient condition for consensus for the delay-free case. Then, based on the obtained conditions for the delay-free case, we deduce an explicit formula for the delay margin of the consensus for the case with time delay by analyzing the relationship between the roots of characteristic equation and the time delay parameter. This leads to the realization that there exists a fundamental tradeoff between consensus performance and robustness to time-delay. We will also provide a more detailed analysis on the consensus condition for the important special case where each agent is a double integrator, and provide a simple and explicit expression for the time delay margin for this case.

2. Problem formulation

2.1. Algebraic graph theory basics

Some basic knowledge on algebraic graph theory is needed for this paper. A multi-agent system (or network) is assumed to have N agents. The communication topology between agents is denoted by a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, where $\mathcal{V} = \{1, 2, \dots, N\}$ is the set of agents, $\mathcal{E} \subset \{(i,j) : i, j \in \mathcal{V}\}$ is the *edge set*, and $\mathcal{A} = [a_{ij}] \in \mathbf{R}^{N\times N}$ is the so-called *weighted adjacency matrix* (or *adjacency matrix*, for short). Each edge (i,j) denotes that agent j obtains information from agent i. The *neighboring set* $\mathcal{N}_i$ of agent i is the set of the agents that can obtain information from agent i. The nonnegative elements and $a_{ij} > 0$ if and only if $i \in \mathcal{N}_i$. The adjacency matrix $\mathcal{A} = \{a_{ij}\}$ is such that each element $a_{ij} > 0$ if $(i,j) \in \mathcal{E}$, or $a_{ij} = 0$. The *in-degree* of agent i is denoted by $d_i = \sum_{j\in\mathcal{N}_i} a_{ij} = \sum_{j=1}^{N} a_{ij}$ and the *in-degree matrix* $\mathcal{D} = \mathrm{diag}\{d_1, d_2, \dots, d_N\}$. The *Laplacian matrix* $\mathcal{L}$ of $\mathcal{G}$ is defined by $\mathcal{L} = \mathcal{D} - \mathcal{A}$. Note that $a_{ij} = a_{ji}, \forall i,j \in \mathcal{V}$ if and only if $\mathcal{G}$ is an undirected graph. A *spanning tree* of a digraph is a directed tree formed by graph edges that connects all the nodes of the graph. It is well known that for an undirected graph, $\mathcal{L}$ is a symmetric, positive semi-definite matrix and all of its eigenvalues are non-negative. Note the special property that $\mathcal{L}\mathbf{1}_N = \mathbf{0}_N$. By denoting all the eigenvalues of $\mathcal{L}$ as $\lambda_i, i = 1, 2, \dots, N$, some properties of the Laplacian matrix are recalled below (Lewis, Zhang, Hengstermovric, & Das, 2014).

Lemma 1. *The Laplacian matrix $\mathcal{L}$ has a simple eigenvalue 0 and all the other eigenvalues have positive parts if and only if the directed network has a directed spanning tree. Specially, for an undirected connected graph, all the eigenvalues of $\mathcal{L}$ are real numbers and can be arranged as $0 = \lambda_1 < \lambda_2 \le \dots \le \lambda_N$.*

We use the following notations and conventions in this paper: R denotes the real number field; $\mathbf{1}_m$ denotes the m-dimensional column vector with all components 1; I_m denotes the m-dimensional identity matrix; $\mathbf{0}$ denotes the zero matrix of appropriate dimension; $\mathrm{Re}(\theta)$ and $\mathrm{Im}(\theta)$ are the real and imaginary parts of a complex number θ, respectively.

2.2. Consensus protocol

In this paper we consider the following general second-order linear dynamic model for each agent $i \in \mathcal{V}$:

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= a x_i(t) + b v_i(t) + u_i(t), \end{aligned} \tag{1}$$

where $x_i(t) \in R$ is the position state, $v_i(t) \in R$ is the velocity state of the ith agent. The initial condition of the agent i refers to $(x_i(0), v_i(0))$.

Remark 2. Apparently, (1) can be seen as $\ddot{x}_i - b\dot{x}_i - a x_i = u_i$, which is a general second-order differential equation. Alternatively, it can be seen as $\dot{\bar{x}}_i = A\bar{x}_i + Bu_i$ with $\bar{x}_i = [x_i, v_i]^T, A = \begin{bmatrix} 0 & 1 \\ a & b \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$,which is the general case of controllable canonical form of second-order dynamics.

Definition 1 (*Second-order Consensus*)**.** A multi-agent system $\mathcal{G}$ with agent model (1) is said to achieve second-order consensus if, for any initial conditions and $i \ne j, i, j = 1, 2, \dots, N$,

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \qquad \lim_{t\to\infty} \|v_i(t) - v_j(t)\| = 0.$$

3. Consensus analysis for the delay-free case

Firstly, we deploy a control protocol without considering the time delay, which is given by

$$u_i(t) = k_1 \sum_{j=1}^{N} a_{ij} \left[x_j(t) - x_i(t)\right] + k_2 \sum_{j=1}^{N} a_{ij} \left[v_j(t) - v_i(t)\right], \tag{2}$$

where $k_1 \in R$ and $k_2 \in R$ are gain coefficients. We define the (composite) state vector $z(t) = [x^T(t), v^T(t)]^T$ with the (composite) position vector and velocity vector $x(t) = [x_1(t), x_2(t), \dots, x_N(t)]^T$, $v(t) = [v_1(t), v_2(t), \dots, v_N(t)]^T$, respectively. The dynamics for the MAS are given by

$$\dot{z}(t) = \Phi z(t), \tag{3}$$

where $\Phi = \begin{bmatrix} \mathbf{0} & I_N \\ aI_N - k_1\mathcal{L} & bI_N - k_2\mathcal{L} \end{bmatrix}$. Define $\hat{x}_i(t) = x_i(t) - x_1(t)$, $\hat{v}_i(t) = v_i(t) - v_1(t), i = 2, 3, \dots, N$, and the state error vector as $\hat{z}(t) = [\hat{x}^T(t), \hat{v}^T(t)]^T$ with $\hat{x}(t) = [\hat{x}_2(t), \hat{x}_3(t), \dots, \hat{x}_N(t)]^T$, $\hat{v}(t) = [\hat{v}_2(t), \hat{v}_3(t), \dots, \hat{v}_N(t)]^T$. We obtain the following error dynamics:

$$\dot{\hat{z}}(t) = \hat{\Phi}\hat{z}(t), \tag{4}$$

where $\hat{\Phi} = \begin{bmatrix} \mathbf{0} & I_{N-1} \\ aI_{N-1} - k_1\hat{\mathcal{L}} & bI_{N-1} - k_2\hat{\mathcal{L}} \end{bmatrix}$, with $\hat{\mathcal{L}} = L_{22} + \mathbf{1}_{N-1}\alpha^T$, and

$$L_{22} = \begin{bmatrix} d_2 & -a_{23} & \cdots & -a_{2N} \\ -a_{32} & d_3 & \cdots & -a_{3N} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{N2} & -a_{N3} & \cdots & d_N \end{bmatrix}, \qquad \alpha = \begin{bmatrix} a_{12} \\ a_{13} \\ \vdots \\ a_{1N} \end{bmatrix}.$$

Apparently, system (1) or (3) achieves consensus if and only if the error system (4) is asymptotically stable.

Let $\beta = [a_{21}, a_{31}, \dots, a_{N1}]^T$, then $\mathcal{L} = \begin{bmatrix} d_1 & -\alpha^T \\ -\beta & L_{22} \end{bmatrix}$. Taking the transformation matrix $S = \begin{bmatrix} 1 & \mathbf{0}_{N-1}^T \\ \mathbf{1}_{N-1} & I_{N-1} \end{bmatrix}$, then we have

$$S^{-1}\mathcal{L}S = \begin{bmatrix} 0 & -\alpha^T \\ \mathbf{0}_{N-1} & \hat{\mathcal{L}} \end{bmatrix}. \tag{5}$$

From (5) we can see that the eigenvalues of $\hat{\mathcal{L}}$ are $\lambda_2, \lambda_3, \dots, \lambda_N$. In order to analyze the asymptotical stability of system (4), we consider its characteristic equation, i.e.,

$$\det(sI_{2(N-1)} - \hat{\Phi}) = \prod_{i=2}^{N} f_i(s) = 0,$$

where

$$f_i(s) = s^2 - bs - a + (k_2 s + k_1)\lambda_i. \tag{6}$$

We obtain the following result.