Automatica 78 (2017) 139–143

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Technical communique

# Active target defense using first order missile models[☆]

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ARTICLE INFO

*Article history:*
Received 21 February 2016
Received in revised form
27 September 2016
Accepted 5 December 2016

*Keywords:*
Optimal control
Missile guidance
Cooperative control

ABSTRACT

In this note the active target defense scenario is analyzed. This entails a Target aircraft being pursued by an Attacker missile while a Defender missile is launched by the Target or by a Target-friendly platform to intercept the Attacker missile. The missiles are modeled using first order dynamics and implement Pure Pursuit guidance laws but are subject to turning rate constraints. The Target's optimal heading is obtained such that the Defender intercepts the Attacker and the terminal Target/Attacker separation is maximized. This work offers more realistic results compared to previous work where simple motion kinematics were used, that is, it was assumed that the missiles are able to turn infinitely fast.

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## 1. Introduction

Pursuit-evasion scenarios involving multiple agents represent important and challenging applications in aerospace control and in robotics. These problems have received increasing attention; for instance, the authors of Sprinkle, Eklund, Kim, and Sastry (2004) employed a receding-horizon approach that provides evasive strategies for an Unmanned Autonomous Vehicle (UAV) assuming a known model of the pursuer's input, state, and constraints. In Earl and DAndrea (2007), a multi-agent scenario is addressed where a number of pursuers are assigned to intercept a group of evaders assuming the dynamics and the goals of the evaders are known. Cooperation between two agents with the goal of evading a single pursuer has been addressed in Fuchs, Khargonekar, and Evers (2010). The work in Scott and Leonard (2013) analyzed a scenario where two evaders search for coordinated strategies to evade a single pursuer but also to keep them close to each other. In Oyler, Kabamba, and Girard (2016) a Prey, Protector, and Predator game setup is used to model rescue missions in the presence of obstacles.

This note is about a three-agent pursuit-evasion engagement. A two-agent team consisting of a Target and a Defender who cooperate is formed; the Attacker is the opposition. The goal of the Attacker is to capture the Target while the Target tries to evade the Attacker and avoid capture. The Target cooperates with the Defender which pursues and tries to intercept the Attacker before the latter captures the Target.

This scenario has been first analyzed in the context of cooperative missile operations in Boyell (1976). Cooperative missile strategies have recently been studied: for instance, in Jeon, Lee, and Tahk (2006) a multi-missile cooperative attack on a stationary target (ship) is considered. Cooperation to control the impact time in order to simultaneously hit the ship is implemented in an outer loop in addition to the typical Proportional Navigation (PN) guidance law. Similar work was presented in Lee, Jeon, and Tahk (2007) for moving targets. The practical application of the Target–Attacker–Defender (TAD) scenario for protection of valuable assets was discussed in Li and Cruz (2011). The authors of Li and Cruz (2011) stated that an optimal evading strategy requires a particular Target heading that makes the Target cross into the reachable set of the Defender (the interceptor) while also accounting for the Attacker strategy. In this paper, we aim at obtaining the optimal Target heading that balances these two goals for the specific case where the missiles employ Pure Pursuit (PP) guidance law.

Different types of cooperation have been proposed in Perelman, Shima, and Rusnak (2011), Rusnak (2005), Ratnoo and Shima (2012) and Rusnak, Weiss, and Hexner (2011) for the TAD scenario. In particular the work in Ratnoo and Shima (2012) and Ratnoo and Shima (2011) considered the Line-of-Sight (LOS) guidance law for the Defender. In those references the Target follows a predetermined trajectory and fires the Defender missile in order to protect itself from the Attacker. The implementation of the LOS guidance law requires the Defender to stay on/ride the LOS

[☆] The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor A. Pedro Aguiar under the direction of Editor André L. Tits.

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http://dx.doi.org/10.1016/j.automatica.2016.12.032
0005-1098/Published by Elsevier Ltd.

between the Target and the Attacker. This strategy was shown to perform well against Attacker missiles with similar or slower speeds than the Defender. However, there exist several situations where the use of the LOS guidance law by the Defender is problematic. For instance, if the Defender is launched by a platform other than the Target such as a wingman or an UAV protecting the Target, then, the Defender may not be able to reach the Target/Attacker LOS and, therefore, will be unable to intercept the Attacker before the latter captures the Target. This situation arises when the Target is not equipped (or runs out) of air-to-air missiles or when a wingman is better positioned to launch the Defender missile. Another case where the LOS guidance law might not be recommended is when the Defender is slower than the Attacker, regardless of which platform launches the Defender. Also, the LOS guidance law requires the Defender to track both the Target and the Attacker or for the Defender to continuously receive data from the Target or a wingman in order to ride on the Target/Attacker LOS.

In this note we consider the scenario where the Attacker and the Defender missiles are hardwired to use PP guidance laws and we determine the Target's instantaneous optimal heading using the theory of optimal control. The gist of the optimization problem faced by the Target: The Target's objective is to lure the Attacker into the path of the Defender while at the same time safeguard itself by evading the Attacker. Hence, the Target balances these two objectives by maximizing the terminal separation with respect to the Attacker at the time of interception of the Attacker by the Defender while ensuring that the Attacker is intercepted by the Defender. The cooperation extended to the Defender by the Target allows a slower and/or distant Defender to successfully intercept the Attacker before the latter reaches the Target. Interception using a less capable Defender has not been considered before and it is obtained here using the cooperative optimal strategy of the Target. This represents the main contribution of the paper.

This note extends the work in Garcia, Casbeer, and Pachter (2015); Garcia, Casbeer, Pham, and Pachter (2014) and Garcia, Casbeer, Pham, and Pachter (2015) where simple motion models for the Attacker and for the Defender were used. In this note we adopt more realistic models for the Attacker and Defender missiles: they can track a commanded heading asymptotically but not infinitely fast. Additionally, we consider the case where the Defender is a fire-and-forget missile. In this case the Defender implements a fixed guidance law to pursue the Attacker and is unable to communicate with another platform. In this situation the Target will maneuver to help the Defender intercept the PP guided Attacker and maximize the terminal Target/Attacker separation.

The note is organized as follows. Section 2 describes the engagement scenario. Section 3 provides the optimal Target heading for the case where the Attacker and Defender missiles implement PP guidance laws. Section 4 provides an example, and conclusions are drawn in Section 5.

## 2. Problem statement

We consider constant speed missiles using PP guidance law while also being subject to turning rate constraints. A representation of the active target defense scenario with missile turning rate constraints is shown in Fig. 1 where the speeds of the Target, Attacker, and Defender are denoted by $v_T$, $v_A$, and $v_D$, respectively. In contrast to previous work (Garcia et al., 2014), the missiles are not able to turn infinitely fast. We now use the following kinematic models for the Attacker and for the Defender missiles

$$\begin{aligned}\dot{x}_i &= v_i \cos\sigma_i\\ \dot{y}_i &= v_i \sin\sigma_i\\ \dot{\sigma}_i &= u_i\end{aligned} \tag{1}$$

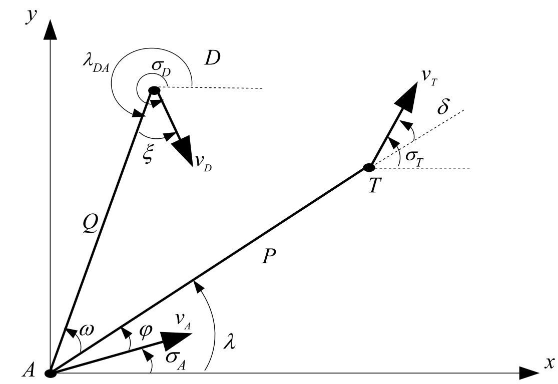


**Fig. 1.** Three-agent aircraft defense scenario.

where the index $i = A, D$ and $\sigma_i$ is the missile's heading angle. The Attacker implements the PP guidance law to pursue the Target. However, since it is not able to turn infinitely fast the following controller is implemented

$$u_A = -a\sigma_A + a\lambda + \dot{\lambda} \tag{2}$$

where $a > 0$ is a constant gain and $\lambda$ is the Attacker's LOS angle. The Defender pursues the Attacker using a similar PP guidance law

$$u_D = -b\sigma_D + b\lambda_{DA} + \dot{\lambda}_{DA} \tag{3}$$

where $\lambda_{DA}$ is the LOS angle of the Defender and, like $a$, $b > 0$ is a constant gain. In this scenario, the Attacker only needs to track the Target and the Defender only needs to track the Attacker. Interception is achieved when the distance between $A$ and $D$ becomes $\bar{Q}$ where $\bar{Q} > 0$ is the Defender's capture radius.

The Target's kinematics are:

$$\begin{aligned}\dot{x}_T &= v_T \cos\sigma_T\\ \dot{y}_T &= v_T \sin\sigma_T.\end{aligned} \tag{4}$$

It is assumed that the Target has complete information: It knows the positions of the Attacker and the Defender and it also knows that both missiles implement PP guidance laws along with the parameters $a$ and $b$. The Attacker reacts to the Target's maneuvers. Knowing this and being a team player, the Target's trajectory is such that the Attacker is pulled into the path of the Defender. This helps the Defender to intercept the Attacker. We are interested in obtaining the Target's optimal heading $\sigma_T$ such that the Defender intercepts the Attacker and the Target survives. However, the Target also needs to protect itself and avoid ending up in the vicinity of the Attacker. Hence, the Target is also striving to maximize the separation between itself and the Attacker when the latter is intercepted by the Defender. In other words, the Target needs to find the optimal heading that trades off the two objectives: lure the Attacker into the Defender's path such that interception is achieved (this is obtained by imposing the terminal constraint in (6) below), and maximize its distance with respect to the Attacker at the time instant of the Defender's interception of the Attacker. The last objective is explicitly considered in the following cost functional

$$J = \sqrt{(x_T(t_f) - x_A(t_f))^2 + (y_T(t_f) - y_A(t_f))^2} \tag{5}$$

where $t_f$ is the time instant when

$$\sqrt{(x_D(t_f) - x_A(t_f))^2 + (y_D(t_f) - y_A(t_f))^2} = \bar{Q}. \tag{6}$$

In order to obtain a compact representation of the three-body dynamics we define the speed ratio parameters $m = v_T/v_A > 0$ and $n = v_D/v_A > 0$. The Attacker is a missile which is faster than