Automatica 74 (2016) 259–269

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Initialization-free distributed algorithms for optimal resource allocation with feasibility constraints and application to economic dispatch of power systems[☆]

Peng Yi[a], Yiguang Hong[b,1], Feng Liu[c]

[a] Department of Electrical & Computer Engineering, University of Toronto, Canada

[b] Key Lab of Systems and Control, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, China

[c] Department of Electrical Engineering, Tsinghua University, China

ARTICLE INFO

Article history:
Received 30 October 2015
Received in revised form
1 April 2016
Accepted 24 July 2016

Keywords:
Resource allocation
Distributed optimization
Multi-agent system
Plug-and-play algorithm
Gradient flow
Projected dynamical system
Economic dispatch

ABSTRACT

In this paper, the distributed resource allocation optimization problem is investigated. The allocation decisions are made to minimize the sum of all the agents' local objective functions while satisfying both the global network resource constraint and the local allocation feasibility constraints. Here the data corresponding to each agent in this separable optimization problem, such as the network resources, the local allocation feasibility constraint, and the local objective function, is only accessible to individual agent and cannot be shared with others, which renders new challenges in this distributed optimization problem. Based on either projection or differentiated projection, two classes of continuous-time algorithms are proposed to solve this distributed optimization problem in an initialization-free and scalable manner. Thus, no re-initialization is required even if the operation environment or network configuration is changed, making it possible to achieve a "plug-and-play" optimal operation of networked heterogeneous agents. The algorithm convergence is guaranteed for strictly convex objective functions, and the exponential convergence is proved for strongly convex functions without local constraints. Then the proposed algorithm is applied to the distributed economic dispatch problem in power grids, to demonstrate how it can achieve the global optimum in a scalable way, even when the generation cost, or system load, or network configuration, is changing.

1. Introduction

Resource allocation is one of the most important problems in network optimization, which has been widely investigated in various areas such as economics systems, communication networks, sensor networks, and power grids. The allocation decisions may be made centrally by gathering all the network data together to a decision-making center, and then sent back to corresponding agents (referring to Ibaraki & Katoh, 1988). On the other hand, differing from this centralized policy, the master–slave-type decentralized algorithms, either price-based (Arrow & Hurwicz, 1960) or resource-based (Heal, 1969), are constructed to achieve the optimal allocations by the local computations in the slave agents under the coordinations of the master/center through a one-to-all communication architecture. However, these methods may not be suitable or effective for the resource allocation in large-scale networks with numerous heterogeneous agents due to complicated network structures, heavy communication burden, privacy concerns, unbearable time delays, and unexpected single-point failures. Therefore, fully distributed resource allocation optimization algorithms are highly desirable.

Distributed optimization, which cooperatively achieves optimal decisions by the local manipulation with private data and the diffusion of local information through a multi-agent network, has drawn more and more research attention in recent years. To circumvent the requirement of control center or master, various distributed optimization models or algorithms have been

☆ This work is supported by Beijing Natural Science Foundation (4152057), NSFC (61333001, 61573344), and Program 973 (2014CB845301/2/3). This work is also partly supported by the National Natural Science Foundation of China (No. 51377092), Foundation of Chinese Scholarship Council (CSC No. 201506215034). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Antonis Papachristodoulou under the direction of Editor Christos G. Cassandras.

E-mail addresses: peng.yi@utoronto.ca (P. Yi), yghong@iss.ac.cn (Y. Hong), lfeng@mail.tsinghua.edu.cn (F. Liu).

[1] Fax: +86 010 62587343.

http://dx.doi.org/10.1016/j.automatica.2016.08.007
0005-1098/

developed (Lou, Shi, Johansson, & Hong, 2014; Nedic, Ozdaglar, & Parrilo, 2010; Sayed, 2014; Yi & Hong, 2014). In light of the increasing attention to distributed optimization and the seminal work on distributed resource allocation in Ho, Servi, and Suri (1980), some distributed algorithms for resource allocation optimization have been proposed in Beck, Nedic, Ozdaglar, and Teboulle (2014), Ghadimi, Shames, and Johansson (2013), Lakshmanan and Farias (2008), Necoara, Nesterov, and Glineur (2011) and Xiao and Boyd (2006).

Continuous-time gradient flow algorithms have been widely investigated for convex optimization after the pioneer work (Arrow, Huwicz, & Uzawa, 1958), and detailed references can be found in Bhaya and Kaszkurewicz (2006) and Liao, Qi, and Qi (2004). Gradient flow algorithms have been applied to network control and optimization (Feijer & Paganini, 2010; Ferragut & Paganini, 2014; Low, Paganini, & Doyle, 2002), neural networks (Liao et al., 2004), and stochastic approximation (Dupuis & Kushner, 1987). Recently, continuous-time gradient flow algorithms have been adopted for solving unconstrained distributed optimization problems (see Droge, Kawashima, & Egerstedt, 2014, Gharesifard & Cortés, 2014, Kia, Cortés, & Martinez, 2015 and Wang & Elia, 2011). Furthermore, the projection-based gradient flow dynamics have been employed for solving the complicated constrained optimization problems in Cherukuri, Mallada, and Cortés (2016), Gao (2003), Nagurney and Zhang (1995), Venets (1985) and Xia and Wang (2000), and the projected gradient flow ideas began to be applied to distributed constrained optimization (see Liu & Wang, 2015, Qiu, Liu, & Xie, 2016 and Yi, Hong, & Liu, 2015).

The economic dispatch, one of the key concerns in power grids, is to find the optimal secure generation allocation to balance the system loads, and hence, can be regarded as a special resource allocation problem. In recent years, there has been increasing research attention in solving economic dispatch problems through a multi-agent system in a distributed manner to meet the ever growing challenges raised by increasing penetration of renewable energies and deregulation of power infrastructure (Cavraro, Carli, & Zampieri, 2014; Zhang, Liu, Wang, Liu, & Ferrese, 2015). Mathematically, this boils down to a particular distributed resource allocation optimization problem. Furthermore, there were various continuous-time algorithms for the Distributed Economic Dispatch Problem (DEDP). For example, Zhao, Topcu, Li, and Low (2014) showed that the physical power grid dynamics could serve as a part of a primal–dual gradient flow algorithm to solve the DEDP, and in fact, it considered physical network interconnections and generator dynamics explicitly, providing a quite comprehensive method and inspiring insights. Moreover, Cherukuri and Cortés (2015) solved the DEDP by combining the penalty method and the distributed continuous-time algorithm in Ho et al. (1980), and proposed a procedure to fulfill the initialization requirement, while Cherukuri and Cortés (2014) constructed a novel initialization-free distributed algorithm to achieve DEDP given one agent knowing the total system loads.

Motivated by various practical problems, including the DEDP in power grids, we study a Distributed Resource Allocation Optimization (DRAO) problem, where each agent can only manipulate its private data, such as the local objective function, Local Feasibility Constraint (LFC), and local resource data. Such data in practice cannot be shared or known by other agents. As the total network resource is the sum of individual agent's local resources, the agents need to cooperatively achieve the optimal resource allocation in a distributed way, so that the global objective function (as the sum of all local objective functions) is minimized with all the constraints (including the network resource constraint and LFCs) satisfied. Note that the LFC is critical for the (secure) operation of practical networks (referring to the communication system in D'Amico, Sanguinetti, & Palomar, 2014 and Johari & Tsitsiklis, 2004 as an example), even though it was not considered in most existing DRAO works. Particularly, for the DEDP in power grids, the generation of each generator must be limited within its box-like capacity bounds. The consideration of LFCs brings remarkable difficulties to existing distributed algorithms designed for the DRAO without LFCs, because the KKT (optimality) conditions for the DRAO with and without LFCs are totally different (referring to Remark 3.3). So far, many DEDP works (such as Cherukuri & Cortés, 2014, 2015, Zhang et al., 2015 and Zhao et al., 2014) have only considered the box-like LFCs. However, the requirement from power industries, such as the secure operation of inverter-based devices in smart grids, promotes the demand to deal with non-box LFCs. This extension is nontrivial, and we will show how to handle it systematically by using projected dynamics in this paper.

Another crucial albeit difficult problem is the initialization coordination among all agents. Many existing results are based on initialization coordination procedures to guarantee that the initial allocations satisfy the network resource constraint, which may only work well for static networks. However, for a dynamical network, the resource has to be re-allocated once the network configuration changes. Therefore, the initialization coordination has to be re-performed whenever these optimization algorithms re-start, which considerably degrades their applicability. Taking the DEDP as an example, the initialization needs to be coordinated among all agents whenever local load demand or generation capacity/cost changes, or any distributed generator plugs in or leaves off (see Cherukuri & Cortés, 2015 for an initialization procedure). This issue has to be well addressed for achieving highly-flexible power grids with the integration of ever-increasing renewables.

The objective of this paper is to propose an initialization-free methodology to solve the DRAO with local LFCs. The main technical contributions of this paper are highlighted as follows:

- By employing the (differentiated) projection operation, two fully distributed continuous-time algorithms are proposed as a kind of projected dynamics, with the local allocation of each agent kept within its own LFC set. Moreover, the algorithms ensure the network resource constraint asymptotically without requiring it being satisfied at the initial points. Therefore, it is initialization-free, different from those given in Ghadimi et al. (2013), Lakshmanan and Farias (2008), Necoara et al. (2011) and Xiao and Boyd (2006), and moreover, provides novel initialization-free algorithms different from the one given in Cherukuri and Cortés (2014).
- The convergence of the two projected algorithms is shown by the properties of Laplacian matrix and projection operation as well as the LaSalle invariance principle. The result can be regarded as an extension of some existing distributed optimization algorithms (Kia et al., 2015; Liu & Wang, 2015; Qiu et al., 2016; Wang & Elia, 2011) and an application of projected dynamics for variational inequalities (Gao, 2003; Xia & Wang, 2000) to the DRAO problem.
- The proposed algorithms can be directly applied to the DEDP in power systems considering generation capacity limitations. It enables the plug-and-play operation for power grids with high-penetration of flexible renewables. Our algorithms are essentially different from the ones provided in Cherukuri and Cortés (2014, 2015), and address multi-dimensional decision variables and general non-box LFCs. Simulation results demonstrate that the algorithm effectively deals with various data and network configuration changes, and also illustrate the algorithm scalability.