Automatica 74 (2016) 308–314

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Distributed adaptive output feedback consensus protocols for linear systems on directed graphs with a leader of bounded input ☆

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ARTICLE INFO

Article history:
Received 23 November 2015
Received in revised form
11 July 2016
Accepted 12 July 2016

Keywords:
Multi-agent systems
Consensus
Output feedback control
Adaptive control
Distributed tracking

ABSTRACT

This paper studies output feedback consensus protocol design problems for linear multi-agent systems with directed graphs containing a leader whose control input is nonzero and bounded. We present novel distributed adaptive output feedback protocols to achieve leader–follower consensus for any directed graph containing a directed spanning tree with the leader as the root. The proposed protocols are independent of any global information of the graph and can be constructed as long as the agents are stabilizable and detectable.

1. Introduction

Over the past decade, the consensus control problem of multi-agent systems has emerged as a focal research topic in the field of control, due to its various applications to, e.g., UAV formation flying, multi-point surveillance, and distributed reconfigurable sensor networks (Antonelli, 2013; Olfati-Saber, Fax, & Murray, 2007; Ren, Beard, & Atkins, 2007). Considerable work from different perspectives has been conducted on consensus and other related cooperative control problems; see the recent works (Antonelli, 2013; Lewis, Zhang, Hengster-Movric, & Das, 2014; Li, Duan, Chen, & Huang, 2010; Olfati-Saber et al., 2007; Ren et al., 2007), and the references therein. Existing consensus algorithms can be essentially divided into two broad categories, namely, consensus without a leader (i.e., leaderless consensus) and consensus with a leader, whereas the latter is also called leader–follower consensus or distributed tracking. In a leader–follower consensus problem, it is often the case that the leader may need to implement its own control actions to achieve certain objectives, e.g., to reach a desirable consensus trajectory or to avoid hazardous obstacles. Compared to leaderless consensus problem, an additional difficulty arises with leader–follower consensus, in which one must deal with the effect of the leader's control input.

A central task in consensus studies is to design distributed consensus protocols based on solely the local information of each agent and its neighbors to ensure that the states of the agents reach an agreement. In most of the previous works on consensus, (e.g., Li et al., 2010, Seo, Shim, & Back, 2009, Zhang, Lewis, & Das, 2011, Yu, Chen, Cao, & Kurths, 2010), the design of the consensus protocols requires the knowledge of certain global information of the communication graph in terms of the nonzero eigenvalues of the corresponding Laplacian matrix, implying that such consensus protocols in essence cannot be determined in a distributed manner. Distributed consensus protocols, nevertheless, can be developed by implementing adaptive laws to dynamically update the coupling weights of neighboring agents, thus removing the aforementioned requirement on the global eigenvalue information. Such adaptive consensus protocols are proposed in Li, Wen, Duan, and Ren (2015), Li, Ren, Liu, and Fu (2013) and Li, Ren, Liu, and Xie (2013b) for linear multi-agent systems. Similar adaptive schemes are presented in DeLellis, diBernardo, and Garofalo (2009) for synchronization of Lipschitz-type complex networks. Note that the

☆ This work was supported in part by the National Natural Science Foundation of China under grants 61473005, 11332001, 61225013, 61528301, and in part by the Hong Kong RGC under the project CityU 111613, CityU 11200415. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Wei Ren under the direction of Editor Christos G. Cassandras.

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http://dx.doi.org/10.1016/j.automatica.2016.07.041

adaptive protocols in DeLellis et al. (2009), Li et al. (2013b) and Li et al. (2013) are applicable to only undirected graphs and the protocols in Li et al. (2015) work for general directed graphs, which however rely on the relative states of neighboring agents. How to design distributed adaptive output feedback consensus protocols using local output information appears much more challenging and remains to be an open issue.

The aforementioned works are concerned with the leaderless consensus problem or distributed tracking problem for the case where the leader has zero control input. The distributed tracking problem in the presence of a leader having a nonzero control input is generally more difficult and has been addressed in Cao and Ren (2012), Dimarogonas, Tsiotras, and Kyriakopoulos (2009), Li et al. (2013b), Li, Liu, Ren, and Xie (2013a) and Mei, Ren, and Ma (2012). In particular, the authors in Cao and Ren (2012) present nonsmooth controllers for first-order and second-order integrators in the absence of velocity or acceleration measurements. The authors in Dimarogonas et al. (2009) and Mei et al. (2012) address a distributed coordinated tracking and containment control problem for multiple Euler–Lagrange systems with one or more dynamic leaders. Distributed static and adaptive protocols are given in Li et al. (2013b) and Li et al. (2013a) for general linear multi-agent systems with a leader of bounded control input. It is worth noting that one common assumption in Cao and Ren (2012), Li et al. (2013b) and Li et al. (2013a) is that the subgraph among the followers is undirected. The case where this subgraph is directed remains unsolved for general linear multi-agent systems. The main obstacle lies in the unpleasant interrelations between the nonlinear functions used to deal with the leader's control input and the directed subgraph among followers.

In this paper, we address the distributed adaptive output feedback consensus protocol design problem for general linear multi-agent systems with directed communication graphs. In this setting, the relative states of neighboring agents are not available, but only local output information is accessible. Note that simply combining the techniques for the state feedback case (e.g., those proposed in Li et al., 2015) and distributed adaptive observer-type protocols (e.g., in Li et al., 2013b) for undirected graphs will not yield a distributed adaptive output feedback consensus protocol applicable to general directed graphs, nor by a routine modification or extension. The main reason is that the monotonically increasing functions introduced in Li et al. (2015), when used for observer-type adaptive protocols in Li et al. (2013b), will still depend on the relative states of neighboring agents.

Partly inspired by the observer structure proposed in Huang (2015), Su and Huang (2012) and Wieland, Sepulchre, and Allgower (2011), in this paper we develop a distributed continuous adaptive output feedback protocol, which includes continuous nonlinear functions to deal with the effect of the leader's nonzero control input. It is shown that the continuous adaptive protocol can ensure the ultimate boundedness of the consensus error and the adaptive gains. The upper bound of the consensus error is explicitly derived, which can be made satisfactorily small by appropriately tuning the design parameters. A distributed discontinuous adaptive output feedback protocol is also presented, to achieve leader–follower consensus for any directed graph containing a directed spanning tree with the leader as the root. The protocols designed in this paper exchange the local estimates among neighboring agents via the communication graph and implement adaptive laws to update the time-varying coupling weights among the agents. As such, these two adaptive protocols use only the local output information and operate in a distributed manner over directed communication graphs. Unlike the protocols in the previous works (Cao & Ren, 2012; Li et al., 2013a,b), the adaptive protocols proposed herein appear to be the first available for linear multi-agent systems with general directed graphs and a leader of nonzero control input.

2. Problem statement

Consider a group of $N+1$ identical agents with general linear dynamics described by

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i, \\ y_i &= Cx_i, \quad i = 0, 1, \ldots, N, \end{aligned} \tag{1}$$

where $x_i \in \mathbf{R}^n$ is the state vector, $y_i \in \mathbf{R}^m$ the measured output vector, $u_i \in \mathbf{R}^p$ the control input vector of the ith agent, respectively, and A, B and C are known constant matrices with compatible dimensions.

The information exchange among the $N+1$ agents is governed by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{0, \ldots, N\}$ is the set of nodes (each node represents an agent) and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the set of edges (each edge represents a communication link between two distinct agents). An edge $(i, j) \in \mathcal{E}$ represents that node i is a neighbor of node j and node j can have access to the output of node i. A directed path from node i_1 to node i_l is a sequence of ordered edges in the form of (i_k, i_{k+1}), $k = 1, \ldots, l-1$. A directed graph contains a directed spanning tree if there exists a root node which has no neighbors but has directed paths to all other nodes in the graph.

Without loss of generality, suppose that the agents indexed by $1, \ldots, N$, are the N followers and the agent indexed by 0 is the leader. Moreover, we assume that the communication graph $\mathcal{G}$ among the $N+1$ agents satisfies the following assumption.

Assumption 1. The graph $\mathcal{G}$ contains a directed spanning tree with the leader as the root node.

Denote by $\mathcal{A} = [a_{ij}] \in \mathbf{R}^{(N+1)\times(N+1)}$ the adjacency matrix associated with the graph $\mathcal{G}$, which is defined as $a_{ii} = 0$, $a_{ij} = 1$ if $(i, j) \in \mathcal{E}$ and 0 otherwise. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbf{R}^{(N+1)\times(N+1)}$ is defined such that $l_{ii} = \sum_{j=0}^{N} a_{ij}$ and $l_{ij} = -a_{ij}, i \neq j$. Under Assumption 1, the Laplacian matrix associated with $\mathcal{G}$ can be partitioned as $\mathcal{L} = \begin{bmatrix} 0 & 0_{1\times N} \\ \mathcal{L}_2 & \mathcal{L}_1 \end{bmatrix}$, where $\mathcal{L}_2 \in \mathbf{R}^{N\times 1}$ and $\mathcal{L}_1 \in \mathbf{R}^{N\times N}$. It is easy to verify that $\mathcal{L}_1$ is a nonsingular M-matrix (Qu, 2009).

Under many circumstances, the leader may need to implement control action to regulate the final consensus trajectory. Therefore, we consider here the general case where the leader's control input u_0 is possibly nonzero and is made available to at most a small subset of the followers. We only assume that u_0 is bounded, i.e., the following assumption holds.

Assumption 2. There exists a constant ω such that $\|u_0\| \leq \omega$.

Our goal is to design distributed output feedback consensus protocols which solve the leader–follower consensus problem for the agents in (1) in the sense of $\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0$, $\forall\, i = 1, \ldots, N$. This design task is generally difficult due to both directed communications among the agents and the effect of the leader's control input.

3. Adaptive output feedback consensus protocols

Based on the relative estimates of the states of neighboring agents, we propose the following distributed continuous adaptive controller to each follower:

$$\begin{aligned} \dot{v}_i &= Av_i + Bu_i + F(Cv_i - y_i), \\ \dot{w}_i &= Aw_i + (d_i + \rho_i)FC\psi_i - \beta Bh_i(B^T Q\psi_i), \\ u_i &= K(v_i - w_i) - \beta h_i(B^T Q\eta_i), \\ \dot{d}_i &= -\varphi_i(d_i - 1) + \psi_i^T C^T C\psi_i, \quad i = 1, \ldots, N, \end{aligned} \tag{2}$$

where $v_i \in \mathbf{R}^n$ is the estimate of the state of the ith follower, $\dot{v}_0 = Av_0 + Bu_0 + F(Cv_0 - y_0)$, $w_0 = v_0$, $\psi_i \triangleq \sum_{j=0}^{N} a_{ij}(w_i - w_j)$,