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Stabilization of linear systems with direct feedthrough term in the presence of output saturation☆

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ABSTRACT

This paper investigates the stabilization problem for controllable and observable linear systems with saturated outputs, in which direct feedthrough term is involved. The necessity of considering the direct feedthrough term is clarified. We present a constructive design strategy to achieve globally asymptotic stability of the systems by output feedback. The controller is continuous during the interval we drive a specific output component out of saturation in order to handle the direct feedthrough term. Moreover, the parameters to be designed for the controller have close relationship with the transfer function and direct feedthrough matrix. Numerical simulations are included to illustrate the effectiveness of theoretical results.

1. Introduction

In practical control systems, the magnitude of input and output signals that actuators and sensors deliver is inevitably limited by physical constrains. If the effects of input and output saturation are not taken into account when we design a controller, peculiar and pernicious consequences may occur.

Unlike input saturation which has been extensively studied in the past decades (see Hu, Lin, & Chen, 2002; Li & Lin, 2015; Ran, Wang, & Dong, 2016; Stoorvogel & Saberi, 2015; Tarbouriech, Garcia, da Silva, & Queinnec, 2011, Zaccarian & Teel, 2011; Zuo, Ho, & Wang, 2010 and references therein), output saturation has received less attention in the literature. Observability of linear systems with sensor saturation was first discussed in Koplon and Sontag (1994). A linear controller was designed to ensure the semi-global stability of the single-input single-output (SISO) linear system with saturated outputs in Lin and Hu (2001) and the multiple-input multiple-output (MIMO) case was subsequently studied in Battilotti (2005). In Kreisselmeier (1996), Kreisselmeier presented a discontinuous output feedback controller to globally stabilize a SISO linear system with output saturation under the condition that the linear system is controllable and observable. The results of Kreisselmeier (1996) have been further extended to the MIMO case in Grip, Saberi, and Wang (2010). Apart from using a single linear or nonlinear controller, anti-windup approach has found an effective strategy for systems with output saturation (see Garcia, Tarbouriech, da Silva, & Eckhard, 2009; Sassano & Zaccarian, 2015; Turner & Tarbouriech, 2009 and references therein).

Nevertheless, all the results mentioned above are limited to linear systems without direct feedthrough from inputs to outputs. In fact, there are many systems with direct feedthrough in the real world. For example, when we transform singular systems which are regular and impulse-free into the state-space form, a direct feedthrough term always appears in the output equation of state-space systems (Dai, 1989). If outputs do not saturate, the effect of direct feedthrough can be easily calculated and eliminated. In such a case, the controller used in linear systems without direct feedthrough can be modified to control the systems with direct feedthrough. However, it becomes quite different and more complicated when the output signals saturate. Owing to the output saturation, we cannot obtain the outputs in the absence of direct feedthrough from the outputs with direct feedthrough even if the input is known. Moreover, a discontinuous controller will lead to discontinuous outputs when direct feedthrough matrix is nonzero. Therefore, it is necessary to consider a particular behavior as a result of discontinuous outputs.

In this paper, we first clarify the necessity of considering the direct feedthrough term in systems with saturated output, which is quite different from the non-saturated case. Such an issue has not yet been taken into account before. The stabilization problem is then investigated for linear systems with saturated outputs,

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in which direct feedthrough term is involved. A novel kind of controller is designed constructively to globally asymptotically stabilize the linear systems with saturated outputs, provided the open-loop systems are controllable and observable. The magnitude of control signal is restricted to prevent specific output jumping over the unsaturation region. In contrast to the controller which can be only used for systems without direct feedthrough in Grip et al. (2010), our designed controller works well whether the direct feedthrough matrix is zero or not because it is continuous during the interval we drive a specific output component out of saturation and the parameters to be designed for the controller have close relationship with the transfer function and direct feedthrough matrix.

2. Main results

Let us consider the following linear systems with direct feedthrough term in the presence of output saturation

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ z &= Cx + Du, \\ y &= sat(z) \end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^m$, $z \in R^p$ and $y \in R^p$ are the state, control input, controlled output and measured output, respectively. A, B, C and D are constant matrices of appropriate dimensions. The direct feedthrough matrix D may not be zero. $sat(z) = [sat(z_1)\ sat(z_2) \ldots sat(z_p)]^\top : R^p \to R^p$ is the standard saturation function with $sat(z_i) = sgn(z_i) \cdot \min\{1, |z_i|\}$ and z_i, $i = 1, 2, \ldots, p$, are elements of z.

Assumption 1. The pair (A, B) is controllable and the pair (C, A) is observable.

Note that there exists a direct feedthrough term in the output equation of (1). Owing to the output saturation, it becomes quite different from the case where systems involve no direct feedthrough. The influences of output saturation are clearly shown as follows:

(i) If the outputs do not saturate, we can design a controller $u = f(\hat{z})$ using the output without direct feedthrough term $\hat{z} = Cx$. Owing to $z = Cx + Du = \hat{z} + Du$, we have $u = f(z - Du)$. Then, the controller based on the output with direct feedthrough is expressed implicitly.

(ii) When saturation occurs in the output channel, however, $\hat{y} = sat(Cx)$ cannot be obtained from $y = sat(Cx + Du)$ since $y \neq \hat{y} + Du$. As a result, it is impractical to design a controller with the form $u = f(\hat{y})$ for systems with direct feedthrough.

The transfer function of (1) is

$$G(s) = C(sI - A)^{-1}B + D. \tag{2}$$

Let $G_i(s)$, $i = 1, 2, \ldots, p$, denote the rows of $G(s)$, i.e.,

$$G_i(s) = C_i(sI - A)^{-1}B + D_i \tag{3}$$

where C_i and D_i are the ith rows of C and D.

In order to construct the controller, we first choose a constant $T > 0$. Then, the controller is constructively designed as $u(0) = 0$ and for any $t \in (kT, kT + T]$, $k = 0, 1, 2, \ldots$,

$$u(t) = \begin{cases} u_l(t), & \sigma_l(kT) = 0, \quad \prod_{i=1}^{l-1} \sigma_i(kT) > 0 \\ u_{p+1}(t), & \prod_{i=1}^{p} \sigma_i(kT) > 0, \end{cases} \tag{4}$$

$$u_l(t) = -\beta_l e^{\rho(t-T_l)} h_l y_l(kT), \quad l = 1, 2, \ldots, p \tag{5}$$

$$u_{p+1}(t) = -B^\top e^{-A^\top(t-kT)} V^{-1}(e^{AkT} W^{-1}(kT)\xi(kT) + \mu(kT)), \tag{6}$$

$$\sigma_i(t) = \int_0^t (1 - |y_i(\tau)|) d\tau, \tag{7}$$

$$V = \int_0^T e^{-A\tau} BB^\top e^{-A^\top \tau} d\tau, \tag{8}$$

$$\mu(t) = \int_0^t e^{A(t-\tau)} Bu(\tau) d\tau, \tag{9}$$

$$W(t) = \sum_{i=1}^{p} \int_0^t (1 - |y_i(\tau)|) e^{A^\top \tau} C_i^\top C_i e^{A\tau} d\tau, \tag{10}$$

$$\begin{aligned} \xi(t) = \sum_{i=1}^{p} \int_0^t (1 - |y_i(\tau)|) e^{A^\top \tau} C_i^\top (y_i(\tau) \\ - C_i\mu(\tau) - D_i u(\tau)) d\tau \end{aligned} \tag{11}$$

where T_l is the instant when $u_l(t)$ is assigned to $u(t)$ for the first time. Because $u(t)$ only switches at time kT, T_l is an integer multiple of T. The scalar ρ satisfies $\rho > \|A\|$. If some elements of $G_j(s)$, $j \in 1, 2, \ldots, p$, are not identically zero, we choose ρ such that $G_j(\rho)$ are nonzero vector. Since the elements of $G_j(s)$ are rational fractions, there always exists a scalar ρ that satisfies the constraint above. h_i, $i = 1, 2, \ldots, p$, are vectors such that $G_i(\rho)h_i > 0$ when $G_i(\rho)$ are nonzero vectors. If $G_i(\rho)$ are zero vectors, we can choose h_i arbitrarily. $\beta_l > 0$, $l = 1, 2, \ldots, p$, are scalars satisfying

$$\beta_l < \frac{1}{e^{\rho T}|D_l h_l|}. \tag{12}$$

Theorem 1. *Consider the linear system (1) that satisfies Assumption 1. Using the controller (4)–(11) with suitable parameters, the origin of (1) is globally asymptotically stable.*

Proof. Let $\sigma_i(t)$ be an indicator characterizing whether $y_i(t)$ is out of saturation. Since $\sigma_i(t)$ is increasing, we can only switch $u(t)$ from $u_i(t)$ to $u_j(t)$, $i < j$.

Step (1) First, we prove that there always exists an integer $\bar{k}$ such that every output component has been out of saturation on $[0, \bar{k}T]$ at least once, i.e. we choose $u_{p+1}(t)$ as $u(t)$ on $(\bar{k}T, +\infty)$. Suppose that it is not tenable. Then, there exists an integer $l \leq p$ such that we choose $u_l(t)$ as $u(t)$ on $(T_l, +\infty)$. In such a case, $y_l(t)$ is continuous on any interval $(kT, kT + T]$ due to the continuity of $u(t)$ on $(kT, kT + T]$. Since $x(t)$ is continuous, using $|y_l(t)| \leq 1$ and (12), we obtain that

$$\begin{aligned} &|y_l(T_l + T^+) - y_l(T_l + T)| \\ &\leq |z_l(T_l + T^+) - z_l(T_l + T)| \\ &= |D_l(u(T_l + T^+) - u(T_l + T))| \\ &= |D_l \beta_l e^{\rho T} h_l(-y_l(T_l + T) + y_l(T_l))| \\ &= \beta_l e^{\rho T}|D_l h_l| \cdot |-y_l(T_l + T) + y_l(T_l)| \\ &< |y_l(T_l + T)| + |y_l(T_l)| < 2. \end{aligned} \tag{13}$$

Since $y_l(t)$ is assumed to be saturated for all time and is continuous on $(kT, kT + T]$, it takes the same value $+1$ or -1 on any specific interval $(kT, kT + T]$. Thus, $y_l(T_l + T^+) = y_l(T_l + T)$, which implies that $y_l(t)$ keeps the same value on $(T_l, T_l + 2T]$. Then, $u(t)$ is continuous on $(T_l + T, T_l + 3T]$, which leads to the same $y_l(t)$ on $(T_l + T, T_l + 3T]$. Therefore, we can deduce recursively that $y_l(t)$ takes the same value $+1$ or -1 on $(T_l, +\infty)$.

For $t \in (T_l + T, +\infty)$,

$$\begin{aligned} &y_l(T_l^+) z_l(t) \\ &= y_l(T_l^+)\,(C_l x(t) + D_l u(t)) \end{aligned}$$