Algebraic estimation and active disturbance rejection in the control of flat systems

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ABSTRACT

This paper proposes a feedback method for the control of uncertain systems with unknown external disturbances, which includes an algebraic estimator and relies on the Active Disturbance Rejection Control (ADRC) approach. The proposed estimator considers a generalized disturbance in order to deal with systems which may simultaneously present time varying parameters, external disturbances, un-modeled dynamics, and process noise. The on-line estimated disturbance is obtained by means of differential algebraic methods and it is used as the major part of an on-line feedback cancellation scheme aiming at linearization and uncertainty suppression. The algebraic estimator proposed in the paper makes unnecessary the use of classical extended state observers, which are widely used in ADRC. The speed of response and reliability of the proposed algebraic disturbance estimator-based control scheme was experimentally tested on three laboratory systems, including a system of directly-coupled DC motors, a roto-magnet system, and a disc and beam system, showing that the experimental results are in excellent agreement with the predictions of the theory.

1. Introduction

There exist several approaches to the problem of unstructured disturbance rejection by means of asymptotic estimation techniques. The work of Johnson (1971) under the name of Disturbance Accommodation Control (DAC) constitutes one of the first references in this area. Johnson has been periodically reviewing the advances of this methodology as presented in Johnson (2008), and has extended it to discrete-time systems (Johnson, 1982) and multivariable systems. A closely related technique is represented by the efforts of Han, summarized in Han (2009), who coined the term "Active Disturbance Rejection Control" (ADRC) for control approaches that decouple the system from the actual disturbances acting on the plant. Gao and his colleagues have continued the work by Han and have popularized ADRC acronym (Gao, 2006; Gao, Huang, & Han, 2001). See also Madoński and Herman (2015) for a recent survey, and Guo and Zhao, (2011, 2013) for a comprehensive treatment of convergence in ADRC.

ADRC is, nowadays, a mature methodology for the control of uncertain systems. Under this approach, the effect of unknown endogenous nonlinearities, along with external (exogenous) disturbances are accurately jointly estimated; this estimation is later used on a suitably canceling control law which renders the partially controlled system linear.

Following a similar approach, Sira-Ramírez and coworkers introduced in Sira-Ramírez, Luviano-Juárez, and Cortés-Romero (2011) an ADRC scheme which uses a Generalized Proportional Integral (GPI) observer with extended states devised to model the lumped perturbation whether of exogenous or endogenous nature. This methodology has been used in several control schemes including laboratory experiments (Coral-Enriquez, Cortés-Romero, & Ramos, 2013; Ramírez-Neria, Sira-Ramírez, Garrido-Moctezuma, & Luviano-Juárez, 2014; Ramírez-Neria, Sira-Ramírez, Luviano-Juárez, & Rodríguez-Ángeles, 2015; Ramos, Cortés-Romero, & Coral-Enriquez, 2015; Sira-Ramírez, Gonzalez-Montanez, Cortés-Romero, & Luviano-Juárez, 2013).

In Fliess and Join (2008) a closely related idea to ADRC, named Intelligent PID Control (IPID) has been proposed. IPID uses Algebraic

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Methods for phenomenological modeling of uncertainties, while replacing the uncertain plant by a first-order or second-order perturbed linear model with free parameters to be on-line identified on the basis of inputs and outputs alone. However, this technique includes a mechanism of re-initialization (see Fliess et al., 2008) to adaptively adjust the free parameters to locally model the uncertain plant. A linear controller is then proposed for the phenomenological model using the on-line gathered information. Our work differs from IPID control in several respects: (1) one retains the known order of the original plant, (2) control gain functions or gain parameters are also retained and, hence, not on-line adapted and (3) the frequency of the reinitialization procedure is reduced by means of time window integration (see Reger, Sira-Ramírez, & Fliess, 2005, 2006).

The disturbance estimation scheme, presented here, is an adaptation of the algebraic parameter identification methodology introduced in Fliess and Sira-Ramírez (2003) (see Sira-Ramírez, Rodríguez-García, Cortés-Romero, & Luviano-Juárez, 2014 for a comprehensive treatment). This method has been developed based on considerations of both, operational calculus (for linear systems), and analysis in the time domain. The contribution of the algebraic method is to obtain a linear set of time-varying integral expressions for the unknown parameters which is independent of the initial conditions and of structural perturbations. The algebraic approach that pursues this work is closely related to the ideas described in Fliess and Sira-Ramírez (2002), Fliess and Sira-Ramírez (2004), Diop and Fliess (1991b) and Diop and Fliess (1991a).

The theoretical foundations of the method, the large number of successful academic examples along with several reports of its use in industrial applications and experimental implementations (Becedas, Feliu, & Sira-Ramírez, 2009; Lafont, Balmat, Pessel, & Fliess, 2015; Menhour, d'Andréa Novel, Fliess, & Mounier, 2013, 2014; Morales, Feliu, & Sira-Ramírez, 2011; San-Millan & Feliu, 2015; Trapero, Sira-Ramírez, & Feliu-Batlle, 2007; Villagra, d'Andréa Novel, Fliess, & Mounier, 2011), encourage us in trying this technique for the disturbance estimation problem in the context of simplified perturbed models, typical of the ADRC approach. The technique performs local piecewise polynomial approximations of the disturbance functions, providing a piecewise continuous update mechanism to keep the approximation valid throughout the evolution of the controlled system. Experimental results are reported on case studies of linear (Becedas et al., 2009; García-Rodríguez, Cortés-Romero, & Sira-Ramírez, 2009; Join, Masse, & Fliess, 2008; Sira-Ramírez, Barrios-Cruz, & Marquez, 2007; Trapero et al., 2007) and nonlinear systems (Cortés-Romero, Luviano-Juárez, Alvarez-Salas, & Sira-Ramírez, 2010). With the aim of not relying on state measurements, the proposed control strategy is only dependent on output measurements.

The main contribution of this article is to propose a combination of an algebraic estimation methodology in the robust feedback control of uncertain systems in the context of output trajectory tracking problems handled by the ADRC approach. Algebraic methods for on-line parameter estimation have been mainly used for fast parameter identification in adaptive-based cancellation of endogenous nonlinearities with the aim of feedback linearization (Sira-Ramírez et al., 2014). The novelty of the work lies in the use of algebraic estimation techniques focused on the context of fast, total, unstructured, disturbance estimation (i.e. estimation of combined effects of endogenous nonlinearities and exogenous disturbances affecting the input-output description of the plant, without concern for the effects of unknown parameters). No efforts are then needed on parameter identification to cancel nonlinear terms of the system by means of feedback linearization. In contrast, the aim here is obtaining a total disturbance identification in order to provide active disturbance rejection through the control law, without the need of parameter estimation or additional state variables information gathering.

The use of algebraic methods in the realm of Active Disturbance Rejection Control (ADRC) has been lacking in the literature. ADRC has been, thus far, characterized by the use of extended state observers and GPI extended observers (Cortés-Romero, Ramos, & Coral-Enriquez, 2014; Sira-Ramírez et al., 2011, 2013; Sun & Gao, 2005; Tian & Gao, 2009). Its use here not only refers to input-output models of differentially flat systems, but the nice integration of the ADRC vision with efficient algebraic methods in the area of, say, total disturbance identification and subsequent adaptative cancellation.

The performance of an Active disturbance rejection control scheme (ADRC) relies on a good estimation of the disturbance, which is usually made in the literature by using high gain observers (Cortés-Romero et al., 2014; Ramírez-Neria et al., 2014, 2015). High gain observers are not convenient in highly noisy systems, due mainly to the noise magnification and peaking phenomena. These drawbacks, if are not properly addressed, may destabilize the system. In contrast to extended state observers, the algebraic estimator introduced in this paper does not suffer of the effects of noise in the same way that the extended observers. In this sense, it is more convenient to be applied in real plants, as has been showed by using several experimental setups.

The proposed control strategy is experimentally validated in three case studies. In the first one, the angular speed of a DC motor is controlled while a current-controlled DC motor injects significant time-varying load torque disturbances. In the second case, the angular position of a DC motor is controlled while a magnetic setup generates load disturbances to the motor. Finally, the third case shows the proposed control strategy performing tracking tasks on a Disc and Beam system.

The rest of the paper is organized as follows. In Section 2 the ADRC control scheme based on algebraic disturbance estimation is explained in detail. In Section 3 the mechanism for on line algebraic estimation of disturbance functions is shown. The procedure is synthesized by a formula that allows for a fast on-line estimation of the effects of additive state dependent nonlinearities and exogenous disturbances, in the form of a time varying signal. Section 4 is devoted to show experimental results that validate our proposed methodology. Finally, some conclusions are given in Section 5.

2. Control based on algebraic estimation of disturbances

Consider a nonlinear, differentially flat, SISO system, given by (Sira-Ramírez & Agrawal, 2004):

$$y^{(n)} = \psi(t, y, \dot{y}, \ldots, y^{(n-1)}) + K_u u + \zeta(t), \tag{1}$$

where u is the input, y is the output, K_u is a constant input gain, and $\psi(.)$ and $\zeta(t)$ are, respectively, endogenous and exogenous perturbations. Note that if there is noise affecting the output or any state of the system, the terms of (1) including noise may be seen as functions of time and included in function ζ as an exogenous perturbation. In this sense, system (1) also models noise acting over any state or output signal.

The objective is to find a control law u for the class of nonlinear system defined by (1), such that the flat output $y(t)$ tracks a given smooth output reference signal $y^*(t)$, in spite of the presence of unknown endogenous and exogenous disturbances represented by $\psi(.)$ and $\zeta(t)$.

In the analysis, it is assumed that the input gain K_u is known and also that the exogenous disturbance $\zeta(t)$ is uniformly and absolutely bounded. In addition, we assume that for any bounded solution $y(t)$ obtained by means of a control input u which is smooth and uniformly bounded, the endogenous additive disturbance $\psi(.)$ is also uniformly absolutely bounded. Uniform and absolute boundedness of perturbations and unknown endogenous injections is a necessary condition to guarantee that the closed loop differential equation (1) exhibits a solution (Gliklikh, 2006; Guo & Zhao, 2011, 2013).

Under these assumptions, and in the spirit of Model Free control, it is conceptually replaced the nonlinear model (1) by a linear perturbed