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Time-optimal flatness based control of a gantry crane

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ABSTRACT

This contribution deals with the flatness based control of a gantry crane, where the control objective is to transfer the load from an initial rest position to a final rest position in a minimal transition time. It is well-known that the type of crane model we consider is a differentially flat system, and that the position of the load is a flat output. We exploit this property both for the design of a tracking control as well as for planning time-optimal reference trajectories for the load. We discuss the design of the tracking control in detail, and show in particular how a standard approach which can be found in the literature can be modified systematically such that instead of measurements of certain time derivatives of the flat output we can use measurements of the state of the system. We also present a new approach for the design of time-optimal reference trajectories. In order to solve the resulting nonlinear optimization problem numerically, we use a primal-dual interior point method. Finally, we conclude with measurement results that stem from an implementation on a laboratory model.

1. Introduction

This paper addresses the flatness based control of a gantry crane which can manipulate a load in a vertical plane. The control objective we consider is to transfer the load between rest positions in a time-optimal way. It is known since about 20 years that this type of crane model is a differentially flat system, with the position of the load as a flat output, see e.g. Fliess, Lévine, Martin, and Rouchon (1995). This property is beneficial both for the design of a tracking control as well as for planning time-optimal reference trajectories for the load. After the mathematical modelling in Section 2, in Section 3 we extensively discuss the design of a flatness based tracking control. We recall two standard methods for the design of flatness based tracking controls, where the first one is based on a dynamic extension of the system (see e.g. Fliess, Lévine, Martin, & Rouchon, 1999) and the second one is based on a quasi-static state feedback (see e.g. Delaleau & Rudolph, 1998). The second method has the advantage that it results in a static control law, but it requires measurements or estimates of certain time derivatives of the flat output. We show that, under certain conditions, this method can be modified systematically such that instead of these time derivatives we can use measurements or estimates of the state of the system, and we demonstrate this for the gantry crane. In Section 4 we present a new approach for the design of time-optimal reference trajectories. In order to solve the resulting nonlinear optimization problem numerically, we use a primal-dual interior point method (IPOPT, see Wächter & Biegler, 2006). For the calculation of the Jacobian matrices which are needed by the solver, we use, amongst others, an implementation based on auto-differentiation (ADOL-C, see Walther & Griewank, 2012). In Section 5 we finally show measurement results from an implementation of our control law on a laboratory model of the gantry crane. Preliminary results of the present work can be found in Kolar and Schlacher (2013).

2. Mathematical modelling

The laboratory model of the gantry crane is shown in Fig. 1. The trolley, which carries a hoist for lifting or lowering the load, is moved by a cable on a rail. The cable is driven by a second hoist, which can be seen in the center of Fig. 1. Both hoists are actuated by gear motors. As long as no external disturbances act on the load, it only moves in a vertical plane, and so we face a planar problem. The mathematical modelling of the gantry crane is based on the schematic diagram shown in Fig. 2. For the modelling we assume that the rope that carries the load is always stretched, which allows us to model the rope with the load as a pendulum (of variable length). Of course, this assumption only holds as long as the vertical acceleration of the load is smaller than the gravitational acceleration. The position of the trolley is denoted by x_T, the rotation angle of the rope drum is denoted by φ, and θ describes the pendulum angle. With the radius R of the rope drum, the length of the pendulum is $l = R\varphi$, and the position of the load is given by $x_L = x_T - R\varphi\sin(\theta)$ and $y_L = R\varphi\cos(\theta)$. The parameter m_T describes the mass of the trolley, and J represents the moment of inertia of the rope drum. The load has the mass m_L, and the gravitational acceleration g points in the positive y-direction. The driving force F which acts on the

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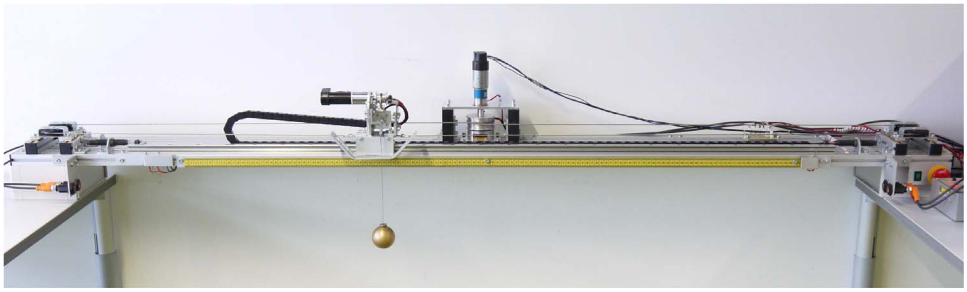

Fig. 1. Laboratory model of the gantry crane.

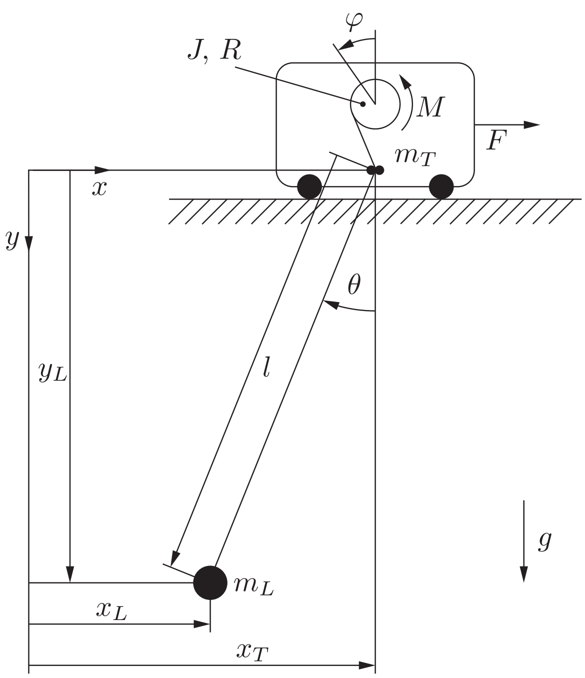

Fig. 2. Schematic diagram of the gantry crane.

trolley and the driving torque M which acts on the rope drum are the control inputs.

Because of the assumption that the rope is always stretched, we can consider the gantry crane as a rigid multi-body system with holonomic constraints. Therefore, the equations of motion can be derived from the Euler-Lagrange equations

$$\frac{\mathrm{d}}{\mathrm{d}t}(\partial_{\dot{q}}T) - \partial_q T + \partial_q V = Q \tag{1}$$

(see e.g. Spong & Vidyasagar, 1989), where T and V are the kinetic and the potential energy of the system, and Q denotes the generalized forces. By q and $\dot{q}$ we denote the generalized coordinates resp. the generalized velocities. With the generalized coordinates $q = (x_T, \varphi, \theta)$, the system's kinetic energy is given by

$$T = \frac{1}{2}m_T\dot{x}_T^2 + \frac{1}{2}J\dot{\varphi}^2 + \frac{1}{2}m_L(\dot{x}_L^2 + \dot{y}_L^2), \tag{2}$$

where

$$\dot{x}_L = \dot{x}_T - R\dot{\varphi}\sin(\theta) - R\varphi\dot{\theta}\cos(\theta)\dot{y}_L = R\dot{\varphi}\cos(\theta) - R\varphi\dot{\theta}\sin(\theta)$$

are the horizontal and the vertical velocity of the load. Note that (2) consists of a term which is due to the translatory motion of the trolley, a term which is due to the rotation of the rope drum, and the kinetic energy of the load. The potential energy of the system is given by $V = -m_L gR\varphi\cos(\theta)$, and the driving force F and the driving torque M result in the generalized forces $Q = (F, M, 0)$. Evaluating (1) results in the equations of motion

$$\begin{aligned}
(m_T + m_L)\ddot{x}_T - m_L R\sin(\theta)\ddot{\varphi} - m_L R\varphi\cos(\theta)\ddot{\theta} + \\
+ m_L\theta(R\varphi\theta\sin(\theta) - 2R\varphi\cos(\theta)) &= F \\
- m_L R\sin(\theta)\ddot{x}_T + (J + m_L R^2)\ddot{\varphi} \\
- m_L R(R\varphi\theta^2 + g\cos(\theta)) &= M \\
- \cos(\theta)\ddot{x}_T + R\varphi\ddot{\theta} + 2R\varphi\theta + g\sin(\theta) &= 0.
\end{aligned}$$

A state representation $\dot{x} = f(x, u)$ is given by

$$\dot{x}_T = v_T\dot{\varphi} = \omega_\varphi\dot{\theta} = \omega_\theta\dot{v}_T = f_{v_T}(\varphi, \theta, \omega_\theta, F, M)\dot{\omega}_\varphi = f_{\omega_\varphi}(\varphi, \theta, \omega_\theta, F, M)\dot{\omega}_\theta = f_{\omega_\theta}(\varphi, \theta, \omega_\varphi, \omega_\theta, F, M) \tag{3}$$

with the state $x = (q, \dot{q}) = (x_T, \varphi, \theta, v_T, \omega_\varphi, \omega_\theta)$ and the input $u = (F, M)$.

3. Flatness based tracking control

For completeness, let us recall that an m-tuple $y = \varphi(x, u, \dot{u}, \ldots, u^{(q)})$ is a flat output of a system

$$\dot{x} = f(x, u) \tag{4}$$

if there exist submersions F_x and F_u and a multi-index $r = (r_1, \ldots, r_m)$ such that locally

$$x = F_x(y, \dot{y}, \ldots, y^{(r-1)})u = F_u(y, \dot{y}, \ldots, y^{(r)}) \tag{5}$$

holds, i.e. x and u can be expressed as functions of the flat output and its time derivatives. This guarantees that the time derivatives of the flat output up to arbitrary order are functionally independent, and consequently (locally) all sufficiently often differentiable trajectories $y(t)$ satisfy the system equations (4). Throughout this paper we will often employ the following notation, which allows to describe in a compact way on which time derivatives of a flat output $y = (y_1, \ldots, y_m)$ (or some other m-tuple of system variables) some function depends. With the multi-indices $a = (a_1, \ldots, a_m)$ and $b = (b_1, \ldots, b_m)$, where a_j and b_j are non-negative integers with $a_j \le b_j$, we use the abbreviation

$$y^{[a,b]} = (y_1^{[a_1,b_1]}, \ldots, y_m^{[a_m,b_m]})$$

with $y_j^{[a_j,b_j]} = (y_j^{(a_j)}, \ldots, y_j^{(b_j)})$, as well as the abbreviation

$$y^{(a)} = (y_1^{(a_1)}, \ldots, y_m^{(a_m)})$$

we have already used above. We also employ the usual conventions $b \pm a = (b_1 \pm a_1, \ldots, b_m \pm a_m)$ and $\#a = \sum_{j=1}^m a_j$ for multi-indices. The map (5) can then be written in the compact form

$$x = F_x(y^{[0,r-1]})u = F_u(y^{[0,r]}). \tag{6}$$

Now it is well-known that the gantry crane is a flat system and that the position of the load is a flat output (see e.g. Fliess et al., 1995). Since $x_L = x_T - R\varphi\sin(\theta)$ and $y_L = R\varphi\cos(\theta)$ only depend on the configuration variables (x_T, φ, θ), the flat output $y = (x_L, y_L)$ is called a configuration-flat output and the system is configuration-flat, see Rathinam and Murray (1998) and Sato and Iwai (2012). However, we want to mention that the gantry crane is not static feedback linearizable (this can be checked by the conditions derived in