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Design of univariate alarm systems via rank order filters

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ABSTRACT

Filtering is one of the techniques used in alarm system design to improve the performance of an alarm system. Due to the fact that the filtered data is no longer independent, computation of the performance indexes (false alarm rate (FAR), missed alarm rate (MAR) and expected detection delay (EDD)) is hard for filters. In this paper, rank order filters are applied in alarm system design. The output of rank order filters is restricted to one of the input samples, thus the probability density function (PDF) of the filtered data can be computed directly from the PDF of the raw data. This feature makes it possible to compute FAR and MAR for rank order filters directly. Further, a method to compute the expected detection delay is proposed for rank order filters despite the dependence of the filtered data. Simulation results shows that the order of rank order filters provide another degree-of-freedom in alarm system design besides the window size, which can be used to improve the alarm performance.

1. Introduction

Alarms are generally used to warn the operators about abnormal events in a process. However, simple alarm configuration may result in alarm floods for operators and reduce its performance as a safeguard for plant operations (Basseville & Nikiforov, 1998; Bristol, 2001; ISA Standard, 2009; Rothenberg, 2009; Yuki, 2002). It is a common practice to compare process variables with alarm limits (also known as trip points or thresholds) to detect possible abnormal events. Whenever a variable exceeds the limit an alarm is raised. Setting of trip points affects the false and missed alarm rates (Izadi, Shah, & Chen, 2010). A false alarm is one that is raised during normal operation of the process, and a missed alarm is one that is not raised during abnormal state of the process. False and missed alarms both give wrong indications of plant states, thus should be reduced as few as possible. As discussed in Izadi, Shah, Shook, Kondaveeti, and Chen (2009), the performance of an alarm system may be measured by the false alarm rate (FAR) and the missed alarm rate (MAR). Both rates are important; a receiver operating characteristics (ROC) curve can be used to measure the accuracy of an alarm configuration by plotting MAR vs. FAR with different trip points for a process variable.

Delay-timers, deadbands and filters are commonly used techniques in alarm system design to reduce false and missed alarms. Delay-timer and deadband techniques are discussed in Adnan, Izadi, and Chen (2011a, 2011b), and the filtering technique is discussed in Izadi et al. (2009) and Cheng, Izadi, and Chen (2011, 2013). It is shown that filtering measured variable before comparing with its alarm limits can significantly reduce FAR and MAR. As an example, Fig. 1 gives the ROC curves of the raw data, the moving average filter, and the optimal nonlinear filter for a process variable assuming the normal and abnormal data follow logistic distributions (Cheng et al., 2011).

From Fig. 1 it is observed that the moving average filter can improve the accuracy of the alarm system compared with the data without filtering. Since the probability distribution functions of the raw data are symmetric and log-concave, the moving average filter is the *optimal* linear FIR filter in this case (Cheng et al., 2011). It is noted that there is a large gap (shadow area) between the ROC curves of the optimal nonlinear filter and the optimal linear filter, that means there are some (nonlinear) filters that may achieve better performance than the optimal linear filter. This motivates us to apply rank order filters in alarm system design.

Rank order filters are nonlinear digital filtering techniques that are widely used in digital image processing to remove noises (Ahsanullah, Nevzorov, & Shakil, 2013; David, 1982; Hardie & Barner, 1994). Rank order filters are robust in environments where the assumed statistics deviate from Gaussian models and are possibly contaminated with outliers. A salient feature of rank order filters is that they restrict the output to be one of the input samples, thus the probability density function (PDF) can be computed directly from the PDF of the raw data. In this paper, we will show that the rank order filters can indeed

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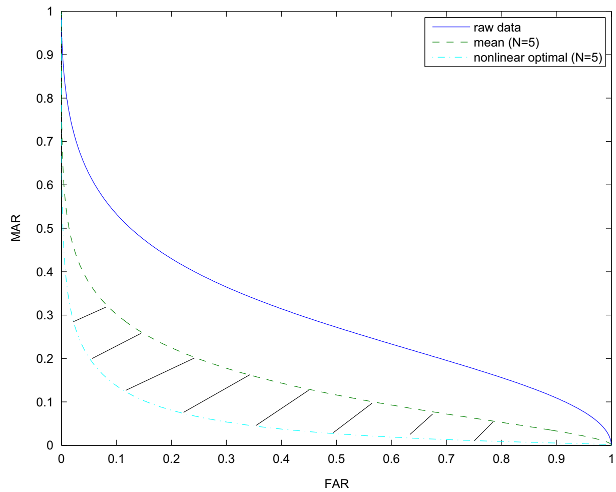

Fig. 1. Performance curves of raw data and filtered data.

achieve better performance than linear filters.

Besides FAR and MAR, expected detection delay (EDD) is also an important index for the performance of an alarm system (Adnan et al., 2011a, 2011b; Xu, Wang, Izadi & Chen, 2012). Detection delay is the difference of time when a fault occurs and when an alarm is raised. Filtering is effective in reducing false and nuisance alarms; however, it will introduce delay in detection of the fault and activation of alarms. Unlike the case for deadbands and delay-timers, filtered data no longer remain independent even if the process data is independent and identically distributed (IID), so the analytical relationship between the filtering parameters and the detection delay is hard to find. Some preliminary discussions are given for the filtering technique in Izadi et al. (2009) based on simulation and no exact quantitative relation is obtained. In Adnan and Izadi (2013), a numerical method is given to compute EDD for moving average filters assuming Gaussian distribution of raw data. Computing EDD for general filters and non-Gaussian distributions is still open, and cannot be found in the literature.

In this paper, a method will be proposed to compute EDD for the rank order filters, noticing that the probability of alarm or no alarm event at any sample time can be computed for rank order filters given the probability of alarm and no alarm events of the raw data, even though the filtered data is not independent. We show that to compute EDD for the rank order filters, the first N sample at the abnormal events are crucial, where N is the window size of the rank order filters.

Remark 1. Throughout the paper, ‘optimality’ means that the following cost function is minimal.

$$J = c_1\,\text{FAR} + c_2\,\text{MAR} \tag{1}$$

where c_1 and c_2 are positive weights for FAR and MAR, respectively. If all the three indices (FAR, MAR, and EDD) are considered, the following cost function is used.

$$J = c_1\frac{\text{FAR}}{\text{FAR*}} + c_2\frac{\text{MAR}}{\text{MAR*}} + c_3\frac{\text{EDD}}{\text{EDD*}} \tag{2}$$

where FAR*, MAR*, EDD* are the (desired) upper bound of FAR, MAR, EDD, respectively, and c_1, c_2, c_3 are positive weights.

The rest of the paper is arranged as follows: Section 2 gives a brief introduction on rank order filters, and Section 3 discusses alarm design via rank order filters to improve the accuracy performance. Detection delay computation is proposed for rank order filters in Section 4. Alarm system design to meet all the requirements on FAR, MAR and EDD via rank order filters is discussed in Section 5 and industrial data are used to demonstrate the design procedure. Finally, conclusions are given in Section 6.

2. Rank order filters

Consider a process variable with sample x_t. Suppose $\{x_1, x_2, \ldots, x_N\}$ are its measurements at sample instant $t = 1, 2, \ldots, N$. The order statistic of rank p is the pth smallest value in the measurements, and is usually denoted as $x_{(p)}$, i.e.,

$$x_{(p)} = \text{rank}_N^p\{x_1, x_2, \ldots, x_N\} \tag{3}$$

where $\{x_{(1)}, x_{(2)}, \ldots, x_{(N)}\}$ are the ordered set of $\{x_1, x_2, \ldots, x_N\}$ such that

$$x_{(1)} \le x_{(2)} \le \cdots \le x_{(N)} \tag{4}$$

There are three special cases for the order statistics:

(1) Largest order: In this case p=N, the output is the maximum of the data set, $x_{(N)}$.
(2) First order: In this case p=1, the output is the minimum of the data set, $x_{(1)}$.
(3) Median: If N is odd, $p = (N+1)/2$, the middle of the ordered samples. If N is even, $p = N/2$ or $N/2 + 1$.

It is easy to verify the following statements: Given a trip point x_{tp},

- $x_{(p)} > x_{tp} \Longleftrightarrow$ at least $N-p+1$ data in $\{x_1, \ldots, x_N\}$ are larger than $x_{tp} \Longleftrightarrow$ at most $p-1$ data in $\{x_1, \ldots, x_N\}$ are less than x_{tp}.
- $x_{(p)} < x_{tp} \Longleftrightarrow$ at least p data in $\{x_1, \ldots, x_N\}$ are less than $x_{tp} \Longleftrightarrow$ at most $N-p$ data in $\{x_1, \ldots, x_N\}$ are larger than x_{tp}.

Given a process variable with sample x_t, a (causal) rank order filter with window size N and order p is defined as

$$y_t = \text{rank}_N^p\{x_{t-N+1}, \ldots, x_{t-1}, x_t\} \tag{5}$$

It is a well-known fact that the cumulative distribution function (CDF) of the filtered data by a rank order filter with size N and order p can be computed as (Ahsanullah et al., 2013)

$$\hat{F}_p(x) = \sum_{i=p}^{N}\binom{N}{p}[F(x)]^i[1-F(x)]^{N-i} \tag{6}$$

and the PDF of the filtered data by the rank order filter is

$$\hat{f}_p(x) = \frac{N!}{(p-1)!(N-p)!}[F(x)]^{p-1}[1-F(x)]^{N-p}f(x) \tag{7}$$

where $F(x)$ ($f(x)$) is the CDF (PDF) of the raw data.

Remark 2.

(1) Delay-timers are closely related to rank order filters. There are two kinds of delay-timers: on-delay and off-delay timers.
   1. When an on-delay timer with size N is applied to a process variable x_t, an alarm is raised at sample t only when there are N consecutive data before sample t larger than the trip point (x_{tp}), which is the same as that the minimum of $\{x_{t-N+1}, \ldots, x_t\}$ is larger than x_{tp}, thus an on-delay timer with size N is equivalent to the minimal filter (rank order filter with window size N and order p=1).
   2. For an off-delay timer with size N, the alarm is cleared at sample t if there are N consecutive data before sample t less than x_{tp}, which is the same as that the maximum of $\{x_{t-N+1}, \ldots, x_t\}$ is less than x_{tp}, thus an off-delay timer with size N is equivalent to the maximal filter (rank order filter with window size N and order p=N).
   3. The above claim is for the high alarm limit. If the low alarm limit is concerned, an on-delay timer is then equivalent to the maximal filter, and an off-delay timer is equivalent to the minimal filter.

(2) The median filters are usually similar to the moving average filters. However, the moving average filters are effective in attenuating